# Composing With Hybrid Microtonalities

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**Abstract:** The history of microtonal music has largely been defined by the creation of closed and self-sufficient musical languages based on systematic derivations from axiomatic principles: for example, Harry Partch's extended just intonation or Ivan Wyschnegradsky's cyclical approach to microtonal equal temperaments. However, contemporary microtonal composers such as Manfred Stahnke and Georg Friedrich Haas often create *hybrid microtonalities* that combine apparently incompatible theories within a single musical work. Various strains of hybrid microtonality are explored through short analytical studies of the author's own compositions, focusing principally on the combination of principles of extended just intonation with various equal temperaments splitting the octave into twelve, nineteen, or twenty-four steps.

**Keywords:** microtonality, just intonation, equal temperament, hybrid microtonalities, tuning, temperament

Much of the rich history of microtonal composition is a narrative of system building and the development of new musical languages—whether an expanded tonality (Harry Partch, Ben Johnston) revitalized through the incorporation of ever more complex frequency ratios, or a set of atonal techniques (as in the later music of Ivan Wyschnegradsky) built on the cyclical and combinatorial potential of microtonal equal temperaments. Not surprisingly, composers have sought to base their new languages on an internally consistent and logical system; Partch, for example, speaks of grounding his music in the "Archean granite" of just intonation.[1] The gains of these systematizing efforts are undeniable, and the search for new languages has brought us such striking and beautiful works as Partch's *Delusion of the Fury* and Wyschnegradsky's *Arc-en-ciel.* Nevertheless, as composers of microtonal music today—more than seventy years since the first edition of Partch's *Genesis of a Music* and eighty years since Wyschnegradsky's *Manuel d'harmonie à quarts de ton*—might we ask whether such closed, circumscribed systems have become as limiting as the legacies of conventional tonality and twelve-tone equal temperament that their creators once sought to escape?

In this article, I explore some of the possibilities of *hybrid microtonalities*, that is, the combination of two or more independent theoretical systemizations within a single musical work. While a loss of theoretical consistency in such hybrids is inevitable, if we are willing to set aside *esprit de système*—that is, the satisfaction of building systems for their own sake—the combination of different theoretical orientations can be the gateway to innovative modes of musical expression.

Hybrid microtonalities can borrow from a wide range of existing theories and techniques. Most frequent are combinations of just intonation and various non-twelve-tone equal temperaments, but other microtonal approaches with potential for hybridization include traditional scales (drawn, for example, from Balinese gamelan or the Arabic *maqam*), spectral harmony (including analysis and resynthesis of harmonic or inharmonic spectra or frequency-based procedures like virtual ring modulation), and split sounds (*Klangspaltung*), the juxtaposition of closely spaced pitches to encourage the emergence of acoustic interference phenomena such as beating.[2]

Examples of works based on hybrid microtonalities include:

- Manfred Stahnke's ***Partch Harp*** (1987–89), which combines extended just intonation tuning of the harp's strings with the 12-TET (twelve-tone equal temperament) modifications of its pedals, along with a non-octaviating equal-tempered scale in the

[1] Harry Partch, *Genesis of a Music: An Account of a Creative Work, Its Roots, and Its Fulfillments*, 2nd ed. (New York: Da Capo Press, 1974), xvii.

[2] These categories of microtonality are based on Georg Friedrich Haas, "Mikrotonalität und spektrale Musik seit 1980," in *Orientierungen: Wege im Pluralismus der Gegenwartsmusik*, ed. Jörn Peter Hiekel (Mainz: Schott, 2007), 138–50.

synthesizer based on a narrowed semitone and allowing the close approximation of just intervals such as 5/4, 7/4, and 11/8.[3]

- György Ligeti's ***Hamburgisches Konzert*** (1998–99/2003), in which the harmonic partials of the horn are modified by transpositions based on 12-TET semitones.[4]
- Georg Friedrich Haas's ***in vain*** (2000), which combines overtone chords in just intonation built on fundamentals separated by equal-tempered semitones (12-TET) to create unusual intervals and complex *Klangspaltung* effects (an effect closely related to Ligeti's writing for horn described above).[5]
- Tristan Murail's ***Winter Fragments*** (2000), in which spectral harmonies and their frequency-based transformations (e.g. ring modulation and spectral interpolation) are rounded off to a grid of equal-tempered quarter tones in the ensemble (24-TET) or semitones in the piano part (12-TET), while the electronics provide exact, non-tempered pitches.[6]
- Hans Zender's ***Bardo*** (2000), which uses 72-TET approximations of frequency-based sum and difference tones as well as 12-TET (winds) and 24-TET (two pianos tuned a quarter tone apart).[7]
- Pascale Criton's ***Chaoscaccia*** (2014), where the radical retuning of the solo cello–four identical strings each separated by just a sixteenth tone (96-TET)—supports a complex gestural language including sonic beatings and oscillations, emergent acoustic phenomena such as combination tones, and noise.[8]

Ultimately, of course, using a particular microtonal theory (hybrid or not) is no guarantee of aesthetic success, just as simply using tonal or twelve-tone techniques does not necessarily produce a meaningful artwork. But I would argue that in each of these works, the composer's adoption of a hybrid approach to microtonal thinking was essential for the unlocking of new intervallic, sonic, and expressive resources. In the following three analytical vignettes, I describe the use of hybrid microtonalities in my own music. In each case, the acoustically based ideal of just intonation is combined with elements of equal temperament. As in the works listed above, I see such hybrids not as regrettable compromises, but rather as pathways to vivid and unexpected soundworlds.

3 Manfred Stahnke, "Meloharmonik," in *Mikrotöne und Mehr: Auf György Ligetis Hamburger Pfaden*, ed. Manfred Stahnke (Hamburg: von Bockel Verlag, 2005), 207–24.

4 Anthony Cheung, "Ligeti's Magic Horn: Parallel Universes of Tuning and Tradition in the Hamburg Concerto" (PhD dissertation, Columbia University, 2010).

5 Robert Hasegawa, "Clashing Harmonic Systems in Haas's *Blumenstück* and *in vain*," *Music Theory Spectrum* 37, no. 2 (Fall 2015): 204–23.

6 Rozalie Hirs, "Frequency-based Compositional Techniques in the Music of Tristan Murail," in *Contemporary Compositional Techniques and OpenMusic*, ed. Bob Gilmore and Rozalie Hirs (Paris: IRCAM, 2009), 93–196.

7 Robert Hasegawa, "*Gegenstrebige Harmonik* in the Music of Hans Zender," *Perspectives of New Music* 49, no. 1–2 (2011): 207–34.

8 Pascale Criton, "Variables, process et degré zéro," *La Deleuziana* 10 (2019): 291–309.

***the clear architecture of the nerves* (2000)**

This short piece for solo horn with piano resonator takes its title from Frank O'Hara's 1952 poem "Early Mondrian":

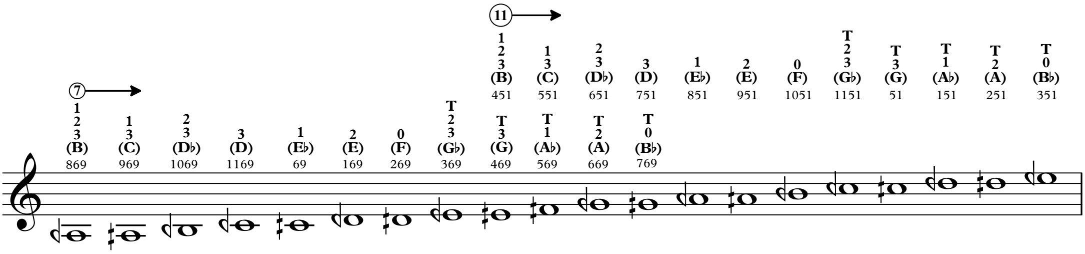
> And
> before us from the foam appears
> the clear architecture
> of the nerves, whinnying and glistening
> in the fresh sun. Clean and silent.[9]

The horn lends itself particularly well to microtones, especially in the upper register. Indeed, the horn embodies a hybrid microtonality in its very design: the valves of the horn (substituting for the historical crooks of the natural horn) adjust the tubing of the instrument to lengths that produce 12-TET fundamentals, while the performer's embouchure selects justly tuned partials from each fundamental's overtone series. Since hornists play much higher in the harmonic series than players of other brass instruments, they regularly encounter "out-of-tune" pitches (with reference to 12-TET) such as the 5th harmonic (14 cents flat from the nearest 12-TET pitch), 7th harmonic (31 cents flat), and 11th harmonic (49 cents flat). In non-microtonal music, these harmonics are typically avoided or corrected by hand or lip to approximate equal temperament.

By embracing these "out-of-tune" pitches rather than avoiding them, the complex hybrid microtonality of the horn becomes an asset rather than a liability. In *clear architecture* I use the 7th and 11th harmonics to allow microtones throughout the upper range of the horn. In the score, these are notated as quarter tones for ease of reading, but in fact the specified fingerings produce a more nuanced result. **Example 1** (written at sounding pitch) shows the fingerings for all microtonal pitches used in the score. "T" refers to the thumb trigger used to switch between the F and B♭ sides of the double horn: thus "T23" is the B♭ horn with valves 2 and 3 pressed, and "2" is the F horn with only valve 2 pressed. While the fingerings based on the 11th harmonic quite closely approximate the tempered quarter tone, those based on the 7th harmonic subdivide the semitone unequally, producing two very different microtonal intervals: 31 cents to the semitone above or 69 cents to the semitone below.[10]

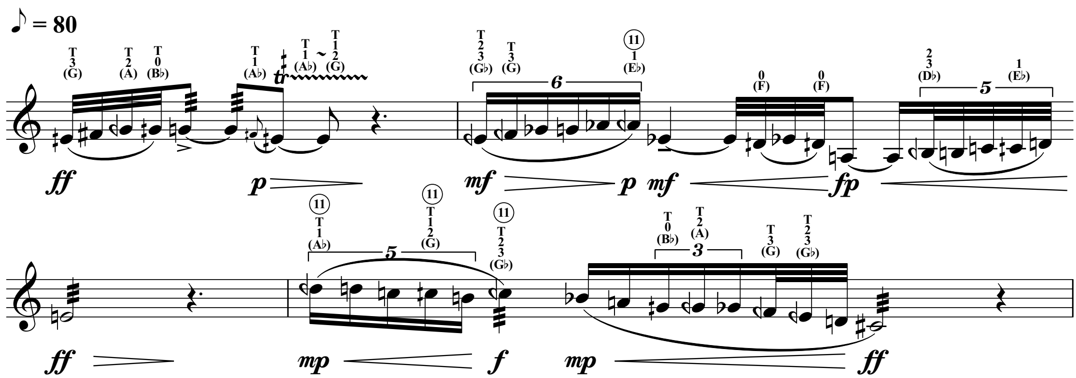
9 Frank O'Hara, *The Collected Poems of Frank O'Hara* (Berkeley: University of California Press, 1995), 37–8.

10 The complexities of horn playing, which often allow several different tunings of a pitch depending on the chosen fingering and harmonic even within the standard notational system, blur this picture slightly since even the "normal" pitches are rarely played in an exact twelve-tone equal temperament. Furthermore, as for all valved brass instruments, combinations of multiple valves (like 123 or 13) tend towards sharpness unless intentionally adjusted flatward. This makes microtones based on these fingerings less reliable, and such combinations are avoided in the piece as much as possible.




**EXAMPLE 1:** Microtonal horn fingering chart for *the clear architecture of the nerves* (2000). At sounding pitch, with pitch classes labeled in cents (C♮ = 0).

The first three measures of the piece are shown in **Example 2**. The hornist (as explained in the score's preface) plays into an open grand piano with the damper pedal held down, as in Luciano Berio's *Sequenza X* (1984) for trumpet. The sympathetic resonance of the strings (to be amplified electronically in larger halls) allows the effect of two or more pitches simultaneously, which became important to the piece's development. Practically speaking, the effect of small distinctions in interval size is much more perceptible when pitches can be heard simultaneously as well as successively.




**EXAMPLE 2:** *the clear architecture of the nerves* (2000), mm. 1–3. Score in C with added fingerings. All microtones based on the 7th harmonic except those marked with an "11" (11th harmonic).

A wide variety of intervals results from the hybrid combination of equal tempered pitches with the 7th and 11th harmonics of equal-tempered fundamentals. For example, measure 2 is based on three different varieties of the "major fourth" (all notated as spanning 5½ semitones): A𝄳 down to E♭ (measure 2) is 551 cents, D𝄲 down to A in the same measure (but now based on the 7th harmonic instead of the 11th) is 569 cents, and B𝄳 up to E is 531 cents. By specifying fingerings, an extensive range of distinct intervals can be produced without overcomplicating the notation. This instrument-specific hybrid microtonality has neither the acoustically pure consonances of just intonation nor the universal transposability of equal-tempered quarter tones, but offers in return a remarkable richness of intervallic color and expression.

### ***Due Corde*** **(2002)**

My duo for two retuned pianos, *Due Corde* (2002), is based on meantone temperament, already a historical hybrid between just intonation and equal temperament. The central idea of meantone temperament is to allow the pure tuning of one interval (usually a major or minor third) through the tempering of another interval (usually the perfect fifth). In certain cases, meantone temperaments can be closely approximated by equal-tempered divisions of the octave, when the tempered fifths complete a full cycle after 31 steps (approximating 1/4-comma meantone) or 19 steps (approximating 1/3-comma meantone, the system used in *Due Corde*).

In Pythagorean tuning, based on perfect 3/2 fifths (702 cents), three stacked perfect fifths produce a minor third (in pitch-class terms) between their endpoints: for example, C–G–D–A yields the minor third A–C. This Pythagorean minor third is considerably narrower (32/27 or 294 cents) than the familiar just minor third (6/5 or 316 cents): the difference between the two is the syntonic comma (81/80 or 22 cents). By reducing the size of each of the perfect 3/2 fifths by one third of this syntonic comma (around 7 cents) to approximately 695 cents, the three fifths now produce a just 6/5 minor third. The major third (at 379 cents) is only 7 cents from the just 5/4 major third of 386 cents, a much better approximation than in equal temperament, with its error of 14 cents. 1/3-comma meantone temperament has its pedigree in sixteenth-century theory (Francisco de Salinas, 1577) but has also been taken up more recently by theorists including Joseph Yasser (1932) and Joel Mandelbaum (1961). A particularly appealing characteristic of 1/3-comma meantone is that it is closely approximated (to within a fraction of a cent) by a 19-fold division of the octave (19-TET), allowing a complex interplay between the acoustically pure minor thirds and sixths and a more atonal, cyclical approach to pitch organization based on equal temperament. The chart below shows the subtle differences between the two tuning systems: as a result of its slightly narrower fifth, the 19-TET circle of fifths returns to its starting point (F♭ or E♯, both 442.11 cents) while the 1/3-comma meantone circle of fifths just misses (by only 0.94 cents) its starting point.

| | *Pitch in 1/3-comma meantone* | *Pitch in 19-TET* |
|---|---|---|
| | Each fifth ≈ 694.79 cents | Each fifth ≈ 694.74 cents |
| | Minor thirds are just (315.64 cents) | |
| F♭ | 441.71 | 442.11 |
| C♭ | 1136.50 | 1136.84 |
| G♭ | 631.28 | 631.58 |
| D♭ | 126.07 | 126.32 |
| A♭ | 820.86 | 821.05 |
| E♭ | 315.64 | 315.79 |
| B♭ | 1010.43 | 1010.53 |
| F | 505.21 | 505.26 |
| C | 0.00 | 0.00 |
| G | 694.79 | 694.74 |
| D | 189.57 | 189.47 |
| A | 884.36 | 884.21 |
| E | 379.14 | 378.95 |
| B | 1073.93 | 1073.68 |
| F♯ | 568.72 | 568.42 |
| C♯ | 63.50 | 63.16 |
| G♯ | 758.29 | 757.89 |
| D♯ | 253.08 | 252.63 |
| A♯ | 947.86 | 947.37 |
| E♯ | 442.65 (0.94 cents higher than F♭) | 442.11 (same pitch as F♭!) |

In *Due Corde,* the nineteen pitches are spread symmetrically between the twelve notes per octave of two retuned pianos; I was introduced to this setup by my colleague, composer and theorist Jonathan Wild. Five of the nineteen pitches of each octave (F–C–G–D–A) are playable on *both* keyboards, while the remaining fourteen are divided between the two instruments (E–B–F♯–C♯–G♯–D♯–A♯ to Piano 1, C♭–G♭–D♭–A♭–E♭–B♭ to Piano II). As a result, each pianist has, in addition to the five common pitches, seven notes available that cannot be played by the other performer. Essential to the concept of the piece is the interaction of the two pianists, sometimes collaborating to create chords together, sometimes concentrating on their own pitches. The division of the pianos into "sharp" (Piano I) and "flat" (Piano II) inscribes the fifth-based conception of the tuning into the performance situation, which is reflected in the piece's moves between "all-sharp-side" and "all-flat-side" pitch collections.

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Piano I: | C | C♯ | | D | D♯ | | E | | F | F♯ | | G | G♯ | | A | A♯ | | B | |
| Piano II: | C | | D♭ | D | | E♭ | | F♭ | F | | G♭ | G | | A♭ | A | | B♭ | | C♭ |
| Cents (C♮ = 0) | 0 | 63 | 126 | 189 | 252 | 316 | 379 | 442 | 505 | 568 | 632 | 695 | 758 | 821 | 884 | 948 | 1011 | 1074 | 1137 |

This tuning offers an increased palette of intervals, some of which are quite unusual in Western music. For example, there are two versions of the semitone: the small chromatic semitone (63 cents) between C and C♯ and the large diatonic semitone (126 cents) between C and D♭. The minor thirds (five chromatic semitones each) are just (316 cents); as a result, four stacked minor thirds do not add up to an octave (as in 12-TET) but rather an augmented octave: e.g, C–E♭–G♭–A♯–C♯! In *Due Corde,* I was particularly attracted to intervals that differ significantly from their 12-TET equivalents. The diminished third or augmented second (e.g. C to D♯) is the unusual interval of 252 cents (4/19 of an octave), and there are two aurally very different "tritones": augmented fourths (568 cents, 9/19 of an octave) and diminished fifths (632 cents, 10/19 of an octave).

The division of pitches between the two pianos inspired me to explore varying degrees of relatedness between the two instrumental parts. At times, the two strands of sound (*due corde*) are closely interwoven, each player contributing the pitches of his piano to a unified effect (a kind of "super-piano"). Certain sections of the piece, however, leave the synchronization of the two strands unspecified, allowing the performers to move independently through their own portions of the tuning system. **Example 3** shows the start of a transition from rhythmic alignment to rhythmic independence. The "all-sharp-side" augmented-second/diminished-third cycle G–A♯–C–D♯–F–G♯ becomes the basis of an ostinato in Piano I at measure 85, accompanying virtuosic, soloistic lines in Piano III (based largely on the different varieties of tritone). Three measures later, the pianos drift out of synchronization, as Piano II continues "with rhythmic freedom" and Piano I independently accelerates. Eventually, both pianos play unaligned solos before finally reuniting over a new ostinato pattern.

The hybrid nature of 1/3-comma meantone or 19-TET offers the composer many different possibilities. While the meantone tuning optimizes the consonance of minor thirds (and their inversions into major sixths), when viewed as an equal temperament it also produces a wide range of new intervals with no counterparts in traditional tonal syntax. In *Due Corde*, I have largely avoided the consonant thirds and sixths, instead taking an essentially atonal approach that values the sonic qualities of the exotic dissonant intervals available in the tuning and the possibility of unusual microtonal interval cycles. The hybridity here is of a quite different nature than that of *the clear architecture of the nerves*: in this case, the meantone and equal-tempered systems are virtually identical to the ear. However, the deployment of the system can emphasize different theoretical aspects of the tunings, creating a more conceptual hybridity: here, the main opposition is between the unusual dissonances best explained by an equal-temperament approach, and the division of the notes between the pianos, which emphasizes the meantone concept of the circle of fifths by sorting the pitches into sharp-side (Piano I) and flat-side (Piano II) groupings. That distinction is woven into the musical fabric of the work through the interactions of the pianists and the contrast between sharp-side and flat-side materials.


**EXAMPLE 3:** *Due Corde,* mm. 85–90

### *it is our tribe's custom to beguile* (2003)

If *Due Corde* takes a historically tonal temperament in an atonal direction, *it is our tribe's custom to beguile* (2003) does the opposite, imposing a tonal order on the usually atonal soundworld of equal-temperament quarter tones (24-TET) by drawing extended analogies with tonal scales, harmonies, and progressions. Hybridity in this case comes from the use of 24-TET both as an approximation of just intonation overtone and undertone chords (Partch's Otonalities and Utonalities) and as an atonal, equal-tempered space that supports the construction of transposable scales based on interval cycles.

As Richard Cohn has observed, many features of Western tonality such as the diatonic scale and consonant triad are overdetermined; that is, they have more than one possible explanation.[11] The diatonic scale has a historical derivation, from the combination of tetrachords with fixed tunings based on simple ratios, but also can be viewed mathematically as a 7-member maximally even set within 12-TET or the product of cyclic generation by perfect fifths. Similarly, the major triad can be explained as both an acoustic phenomenon (partials 1–6 of a harmonic series) and through its embeddings in the diatonic and chromatic scales (as well as theoretical conceptions like the *Tonnetz*). These multiple explanations of tonal materials suggest an inherent hybridity in tonality itself.

The hybrid microtonality of *it is our tribe's custom to beguile* takes scale construction as its starting point, mimicking the generation of the major scale through a cycle of perfect fifths but replacing the fifth with the microtonal interval of 3½ semitones (7 quarter tones). An incomplete cycle of 3½-semitone intervals yields a 17-note scale:

G B𝄳 D F𝄲 A C𝄲 E G𝄲 B D𝄲 F♯ A𝄲 C♯ E𝄲 G♯ B𝄲 D♯

Below, the 17 pitch classes are rearranged in ascending order starting on D, with gaps added to show the larger intervals of one semitone. Note that the scale falls into groupings of 3 and 2 adjacent notes.

D D𝄲 D♯ E E𝄲 F𝄲 F♯ G G𝄲 G♯ A A𝄲 B𝄳 B B𝄲 C𝄲 C♯

Stopping the cycle at the 17th note allows this scale to imitate one of the central features of the diatonic scale. Just as a C major scale can be transformed into one of its nearest neighbours (G major or F major) by the shift of just one pitch by a semitone up or down, this scale starting on D can be transformed into a transposed version on B𝄳 by raising D♯ to E𝄳, or to a transposed version on F𝄲 by lowering G to G𝄳. Besides this 17-note scale, several other scales that transform in a similar way can be constructed in the quarter-tone world: these have 5, 7, 11, 13, or 19 notes. The 13-note scale is Ivan Wyschnegradsky's *échelle chromatique diatonisée*, a "diatonicized" version of the chromatic scale based on a generating interval of 5½ semitones.[12] I found the 17-note scale particularly appealing: it allows complex melodies while retaining a recognizable aural distinction between the notes in the scale and those excluded.

The 17-note scale has the added benefit of coinciding with chords derived from the overtone series. I took Rameau's concept of the major triad as partials 1–6 of a resonating *corps sonore* and expanded it: my basic harmonic unit is an 8-note chord, built by rounding

[11] Richard Cohn, "Neo-Riemannian Operations, Parsimonious Trichords, and Their *Tonnetz* Representations," *Journal of Music Theory* 41, no. 1 (1997): 1–66; Richard Cohn, "Music Theory's New Pedagogability," *Music Theory Online* 4, no. 2 (1998), https://mtosmt.org/issues/mto.98.4.2/mto.98.4.2.cohn.html.

[12] The relevant mathematics are discussed in David Lewin, "Cohn Functions," *Journal of Music Theory* 40, no. 2 (1996): 181–216.

off partials 8 to 15 to the nearest quartertone. This is a relatively rough approximation, especially for partials 5 and 10 (386 cents above the fundamental in pitch-class terms), partials 7 and 14 (969 cents), and partial 13 (841 cents), which are a considerable distance from the nearest tempered quarter tone. The rounding errors are compounded in intervals where partials are rounded in opposite directions: for example, when partial 13 is rounded from 841 cents up to 850 and partial 14 is rounded down from 969 cents to 950, the interval between the approximations is only 100 cents, while the just-intonation interval between partials 13 and 14 is 128 cents. Despite these relatively poor approximations, the possibilities of intervallic reinterpretation and harmonic pivots offered by 24-TET offer an appealing trade-off: for example, the interval of 150 cents can, depending on the harmonic context, be heard as the interval 11/10, 12/11, 13/12, 14/13, or even 15/14 (with a substantial deviation of 31 cents from the just interval). I would argue that in a sufficiently clear harmonic context, these approximations of the partials do not hide their functional identity, just as when we accept tonal music played in equal temperament (another hybrid situation) as approximating just consonances.[13] And, as in tonal music, performers can adjust their playing to bring these approximations closer to their just-intonation equivalents.

These 8-note chords approximating partials 8–15 fit neatly into my 17-note scale; every 17-note scale contains three instances of this type of chord, each separated by the interval of 3½ semitones. This is again analogous to the tonal system, where three major chords separated by fifths (tonic, dominant, and subdominant) fit into each major scale.

Within the D scale, one can find 8-note overtone chords on the following pitches:

| **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** |
|---|---|---|---|---|---|---|---|
| F𝄲 | G𝄲 | A𝄲 | B | C𝄲 | D | D♯ | E𝄲 |
| D | E | F♯ | G𝄲 | A | B𝄳 | B𝄲 | C♯ |
| B𝄳 | C𝄲 | D𝄲 | E | F𝄲 | G | G♯ | A𝄲 |

And, taking a cue from nineteenth-century harmonic dualists like Hugo Riemann and Arthur von Oettingen (as well as Harry Partch, of course), "minor" or "undertone" versions of these chords can be built by measuring identical intervals downward from the initial pitch. Like the overtone chords, three undertone chords fit into each transposition of the scale:

[13] For an extended version of the argument that listeners interpret intervals as representations of the nearest and simplest just-intonation equivalents, see Robert Hasegawa, "Just Intervals and Tone Representation in Contemporary Music," PhD dissertation, Harvard University, 2008. Composer/theorist James Tenney makes a similar argument for a "tolerance range" within which we accept mistuning of just intervals, though I suspect that Tenney would find the quarter-tone tunings used here too imprecise, even as approximations. James Tenney, "The Several Dimensions of Pitch," in *The Ratio Book: A Documentation of the Ratio Symposium, Royal Conservatory, The Hague, 14–16 December 1992*, ed. Clarence Barlow (Cologne: Feedback Studio Verlag, 2001), 110.

| **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** |
|---|---|---|---|---|---|---|---|
| B𝄲 | A𝄲 | G𝄲 | F♯ | E𝄲 | D♯ | D | C𝄲 |
| G♯ | F♯ | E | D𝄲 | C♯ | B𝄲 | B𝄳 | A |
| E𝄲 | D𝄲 | C𝄲 | B | A𝄲 | G♯ | G | F𝄲 |

The inclusion of these six overtone- or undertone-based harmonies in each scale offers another close parallel to the tonal system, with major and minor "tonics," "dominants," and "subdominants" for every "key." Formulating the system in this way allows me to construct quarter-tone analogies for many of the hierarchical structures of tonal music. I decide on a chord that controls a given stretch of time; within this chord, neighbor notes (either within the scale or "chromatic neighbors" from outside it) can embellish chord tones, passing tones can move from one chord tone to another, a voice can arpeggiate notes of the harmony, etc. The harmony can progress to any of the other chords of its scale, or even modulate to a new scale. It is even possible to imagine quarter-tone prolongations: for example, the passing tone G♯ between the G𝄲 and A of the D overtone harmony could be "composed out" into a G♯ overtone or undertone chord. In the excerpt of the piece shown in **Example 4,** the "tonic major" chord D moves to a "minor" chord descending from b𝄲, then to "major" chords on B𝄳 and finally F𝄲, symmetrically framing the central D. These harmonies all fit within the same D scale, but as one harmony progresses to the next, different pitches become consonant and dissonant in relation to the prevailing chord, and the functional relationship of each pitch to the chord "root" changes.

**Why hybrid microtonality?**

One of the great rewards of composing in these three very different hybrid systems has been discovery: the challenge in each new piece of learning to hear, audiate, and invent in a new soundworld with its own resources, shortcomings, and affordances. Rather than setting up a system once and for all and then writing further pieces within that system, hybridity encourages the constant posing of new problems, acting as both compositional constraints and spurs to the imagination. This is an instance of "problem finding" as described by Getzels and Csikszentmihalyi in their study of the artistic process: "the formulation of a creative problem is the forerunner of a creative solution."[14] Being forced to rethink the basics of musical language for each piece, even to the extent of customizing a tuning for a particular instrument (as in *the clear architecture of the nerves*), is a continuing catalyst for creativity.

While I continue to write pieces that draw on more "pure" systems of microtonality (such as *Chaconne for James Tenney* in extended just intonation or *Ajax is all about attack 1* in

[14] Jacob Getzels and Mihaly Csikszentmihalyi, *The Creative Vision: A Longitudinal Study of Problem Finding in Art* (New York: John Wiley & Sons, 1976), 4–5.

equal-tempered quarter tones), the appeal of microtonal hybrids is worth reiterating here, particularly as they may seem ad hoc or ephemeral compared to more established and logically grounded systems. Purity—whether of theoretical principles or sonority—cannot be an end in itself, and artistic results, not systematization for its own sake, should be the test of any compositional approach to microtones. From the present-day perspective on a long and rich history of microtonal music, we can leave the polemics between proponents of different systems behind and take what is valuable from each, accepting the inevitable clashes and unpredictable combinations as assets rather than drawbacks. Among the charms of hybrid microtonality are the strange and mysterious intervals that result from the interaction of systems. As we try to make aural sense of them, they remain ambiguous, chimeric, and resistant to easy explanation. For me, the ongoing attraction of hybrid microtonality—though it offers neither the pure-ratio consonances and logical clarity of extended just intonation nor the mathematical regularity and ease of transposition and transformation of equal temperament—lies in the fresh sounds (and novel challenges) of each new project.


**EXAMPLE 4:** *it is the custom of our tribe to beguile*, mm. 1–15

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