A Rational Intonation Approach to Persian Music

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Abstract: The correct intonation and size of intervals in Iranian classical music (Radif) has been debated since the beginning of the 20th century and has led to much discussion and produced differing opinions. In an attempt to arrive at a systematic approach, Iranian and Western theorists and musicologists have put forward several different theories.

As a composer who has always been inspired by Iranian classical music, I have been paying more attention to tuning and intonation in recent years. I have been using a rational intonation approach to create my own sonic world. In this article, I present my approach to rational intonation for Iranian classical music.

In order to achieve my goal, I first outline the methods used by other researchers working on this topic. By demonstrating the limitations of the methods used until now, I present my view on this subject in the second part. I also describe the method I have used to obtain intervals for a detailed interval palette in Iranian music. In this way, I can create and notate Iranian classical music without losing the exquisite nuances of the music.

This is a personal approach to intervals in Iranian music that can be used both in the composing and notating of classical Persian music.

Keywords: Just intonation, Persian classical music, Radif of Iranian music, Mediaeval tuning systems, historical temperaments, interval, intonation, microtones, pure interval, tuning, tuning adjustment

Introduction

As an Iranian composer, I have always composed music inspired by the classical music of my homeland known as "Radif". In short, Radif is a complex network of smaller melodic structures with or without specific rhythmic patterns, divided into twelve groups by their modal relationships; each of them is called a *dastgah* (plural *dastgah-ha*). Depending on the school or instrument, a *dastgah-ha* may contain hundreds of melodic structures called *gusheh* (plural *gusheh-ha*).

As someone fascinated by physics and mathematics, both psychoacoustics and microtonality are very attractive sources of inspiration. My interest in microtonality encompasses all kinds of approaches: different temperament systems, just intonation techniques, non-octave subdivisions or the so-called traditional non-Western practices. Nevertheless, my focus on Iranian microtonality is primarily on the tuning systems written by Persian polymaths between the 9th and 15th centuries, with an emphasis on the works of Fārābi (10th century), Ibn Sinā (10th century) and Safiaddin Ormavi (13th century).

During my research on the mediaeval tuning systems of Iran, I have found similarities between these systems and the contemporary classical music of Iran, Radif, which has long been my source of inspiration. Working with ratios and focusing on rational intonation in my own compositions showed me a way to translate Radif into the language of ratios. In addition to analysing various theories of contemporary researchers aimed at finding the "true" size of intervals in Persian music, I would like to propose my gamut and a palette of rational intervals derived from the harmonic series, which I believe can help both ethnomusicologists and composers in creating new music or notating the existing classical music of Iran.

Intervals of Iranian classical music

Since the beginning of the 20th century and the acquaintance of Iranian scholars with Western music theory and notation systems, there have always been discussions and differing opinions about the "true" intonation and size of intervals in Iranian classical music. The first scholar to notate and transcribe the Radif and write on Iranian music theory was Ali-Naqi Vaziri (1886–1979). *The Harmony of the Music of Iran or Quarter-tone Music* (1935), written by Vaziri, was the first attempt in Iran to find a solution for the harmonisation of Iranian monophonic modal music. In the book, Vaziri describes the intervals that do not exist in the music of the Western world and explains that they are similar to the components of the harmonic and subharmonic series. Because of his great interest in the Europeanisation of Iranian music theory, Vaziri introduces the idea of tempered intervals from Western music theory and suggests that we can use the same approach for microtones in Iranian classical music. Based on this analogy, he proposes a gamut of 24 equally spaced microtones per octave, with a tempered size of 50¢ for each quarter tone.

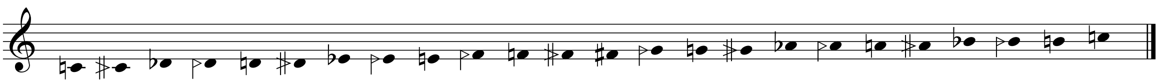
To complement this system, he also developed two accidentals for the notation of the microtones of classical Iranian music. Scholars and musicians who study and perform Iranian music around the world continue to use these accidentals.

Below is an illustration of "Sori" and "Koron", the two accidentals used to distinguish the microtone of classical Iranian music.

𝄲 – Sori, raises the pitch by a quarter tone

𝄳 – Koron, lowers the pitch by a quarter tone

Many Iranian scholars and musicians still accept Vaziri's 24-note EDO scale as the main notation system. In reality, however, they do not tune their instruments to 24 equally spaced quarter tones per octave, so Vaziri's method is not so "true" when it comes to intonation. Later in this article, I will show that the intervals of Iranian classical music are not divided into quarter tones but actually vary in size.



The 24-tone EDO scale of Vaziri

Mehdi Barkeshli (1912–1988) was a scholar who devoted his life to researching the true size of intervals in Iranian classical music. He rejected Vaziri's equally divided gamut and argued that the true size of intervals in Iranian music is neither quarter tones nor a third of a tone, as some Western theorists suggested in the late 19th century. Rather, it lies "somewhere" between a quarter tone and a third of a tone.

Barkeshli suggested that we find the true Iranian intonation in the treatises of Iranian polymaths, especially those of **Abu Nasr Fārābi** (870–950) and **Safiaddin Ormavi** (d. 1294). A physicist by profession, he also measured the size of intervals by examining and analysing the recordings of masters of Iranian classical vocal music. He found that the size of the whole tones and semitones in classical Iranian music is almost identical to the Pythagorean tuning system, based on his findings from the mediaeval treatises and studying the interpretations of the Radif by his contemporaries. Therefore, a tetrachord, e.g. of the mode "Mahur", structured on the basis of the whole tones and semitones, should look as follows:

♮C – ♮D – ♮E – ♮F or as he mentions in cents 205¢ + 204¢ + 89¢ = 498¢

As a result of his investigations, Barkeshli also found three different sizes of ♭D and ♭E in the structure of the tetrachord of other Iranian modes; he described the size of these intervals as follows:

$\flat D^1 = 89¢$, $\flat D^2 = 119¢$, $\flat D^3 = 181¢$ from ♮C

$\flat E^1 = 89¢$, $\flat E^2 = 119¢$, $\flat E^3 = 181¢$ from ♮D

Barkeshli correlates the interval between ♮C and $\flat D^1$ (89¢), which is an identical interval in the mode of "Chahargah", as a Pythagorean limma or "Zaed" of Safiaddin Ormavi with the ratio of $\frac{256}{243}$ (90¢). He goes on to describe the interval between ♮C and $\flat D^2$, with the size of 119¢ and the ratio of $\frac{15}{14}$ as the unique interval of Iranian classical music, that exists in the mode of "Dashti". Barkeshli compares these intervals with the Pythagorean apotome ($\frac{2187}{2048}$), with a small difference of 6¢ or the melodic distance of 5120:5103. He adds that the interval between ♮C and $\flat D^3$ with a size of 181¢ is not very common in contemporary Iranian classical music but could be compared with the "Mojannab" of Safiaddin Ormavi, which combines two Pythagorean limmas. At the final stage of his research, Barkeshli describes the group of ♭E intervals by concluding that, for Safiaddin Ormavi, the $\flat E^1$ substitutes "Wusta Furs" and the interval between ♮D and $\flat E^2$ are the same intervals – just like ♮C to $\flat D^2$ – with the ratio of $\frac{15}{14}$. And finally, the interval of $\flat E^3$ can be recognised as "Wusta Zalzal" of Safiaddin Ormavi.

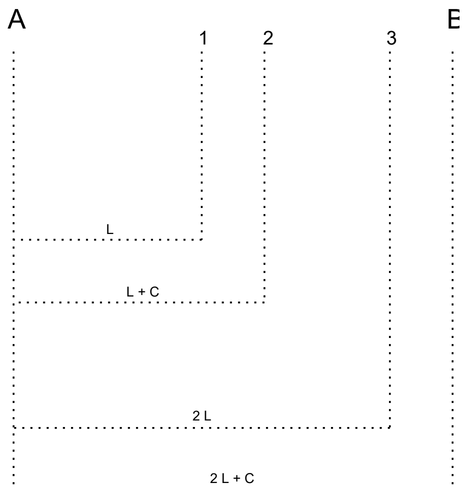
	♮C	**$\flat D^1$**	**$\flat D^2$**	**$\flat D^3$**	**♮D**
Barkeshli	0	89¢	119¢	181¢	205¢
Pythagorean	1/1	Limma 256/243	≈Apotome 2187/2048	Diminished third 65536/59049	9/8
Ormavi	Motlagh 1/1	Zayed 256/243	15/14	Mojannab 65536/59049	Sabbabeh 9/8

	$\flat E^1$	**$\flat E^2$**	**$\flat E^3$**	**♮E**	**♮F**
Barkeshli	294¢	324¢	386¢	409¢	498¢
Pythagorean	32/27		Diminished fourth 8192/6561	81/64	4/3
Ormavi	Wusta Furs 32/27	135/112	Wusta Zalzal 8192/6561	Bensir 81/64	Khensir 4/3

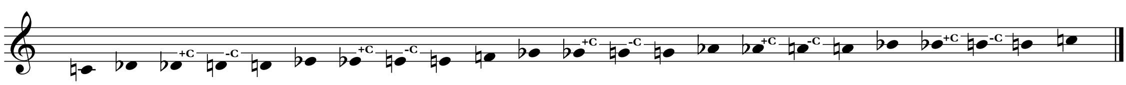
Division of the Tetrachord, as described by Barkeshli

Based on these descriptions and his tendency to find a type of temperament following Safiaddin Ormavi's theories and findings, Barkeshli suggests that the internal structure of a whole tone in classical Iranian music should be created by combining Pythagorean limma and comma. Considering the sizes of the Pythagorean limma and comma ($\frac{256}{243}$ and $\frac{531441}{524288}$), he shows the subdivision of a whole tone as follows:

L, L+C, L+L, L+L+C (90, 113.7, 180.4, 203.9)



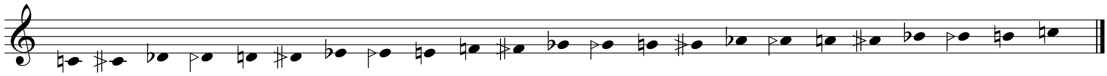
Based on this structure, Barkeshli proposed a 22-tone gamut that was not evenly divided:



As a scholar, he uses the verbal information above the standard accidentals to show the true size of intervals, a confusing and impractical method for performers. To improve notational practice, in his book *Radif of Persian Classical Music, collected by Musâ Ma'rufi* (1963/2011), Barkeshli uses Vaziri's accidentals for his 22-tone gamut, with a precise definition of the size of the deviation that these accidentals produce:

– Sori raises the pitch by a Pythagorean comma ($\frac{531441}{524288}$ or 23.5¢)

– Koron lowers the pitch by a Pythagorean limma ($\frac{256}{243}$ or 90¢)

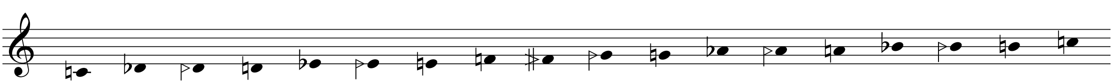


The 22-tone scale of Barkeshli

The Dastgah Concept in Persian Music (1990) by **Hormoz Farhat** (1928–2021) is one of the most reliable sources for musicians studying classical Iranian music both inside and outside Iran. In his book, Farhat rejects the two earlier theories of 24-tone and 22-tone per octave. He explains that both Vaziri and Barkeshli were influenced by Western music theory and tended to unnecessarily adopt Western theoretical concepts, and Farhat completely rejects their idea that Iranian music is based on a scale as in Western classical music. He adds that Barkeshli's analogy for creating a scale based on the ideas of the mediaeval Iranian polymaths is impractical and inapplicable to contemporary practice. Furthermore, Farhat objects to Barkeshli's method of analysing intervals in Iranian classical music based on vocal repertoire. For Farhat, this was a big mistake owing to the voice's unstable nature. Therefore, it cannot be reliably used for the analysis of intervals.

Based on these arguments and his own studies and research, Farhat introduces the concept of "flexible intervals" in Persian music. In contrast to Barkeshli, who focuses exclusively on the vocal repertoire of Iranian classical music, Farhat examines instruments with fixed frets on the fingerboard, such as the tar and the setar, and explains there are no fluctuations during performance because of the fixed frets on these instruments.

As a result of his investigation, the minor second and the whole tone were shown to remain relatively stable when analysed in different modes and by different interpreters, and were show to be similar to the Pythagorean limma ($\frac{256}{243}$ or 90¢) and the Pythagorean whole tone ($\frac{9}{8}$ or 204¢). The intervals between them, however, are not fixed. Farhat introduces two intervals of "small neutral tone" with flexible sizes between 125 and 145¢ and a "larger neutral tone" between 150 and 170¢ and suggests using the mean of these sizes, 135 and 16¢, respectively. Based on these intervals and the fact that the fret system of the two instruments – tar and setar – has only 17 notes per octave, he introduces the 17-tone non-equal gamut as follows:



The 17-tone scale of Farhat

♮C – ♭D – 𝄳D – ♮D (90¢ – 45¢ – 70¢),
the same intervals repeat between ♮D–♮E, ♮G–♮A and ♮A–♮B.
♮F – 𝄲F – 𝄳G – ♮G (65¢ – 65¢ – 70¢)

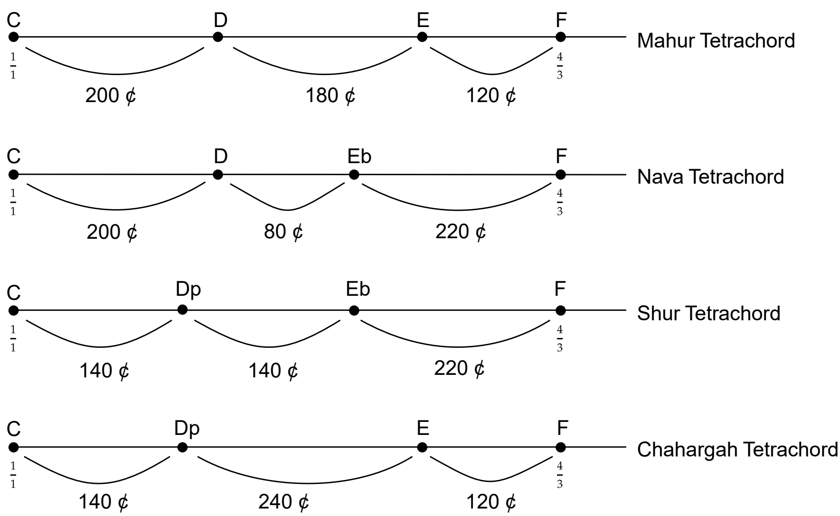
What is more, no other pitches exist between ♮E – ♮F and ♮B – ♮C (in both cases 90¢)

He also introduces the "plus tone" (270¢). He adds that the plus tone is a very unstable interval, larger than the whole tone but not as large as the augmented tone ($\frac{19683}{16384}$ or 318¢). It is only found in a small number of modes and is always preceded by the small neutral tone.

Although all contemporary Iranian theorists before Farhat have attempted to theorise Iranian music in a Western way, comparing the Iranian modal system with Western 12-tone scales, Farhat rejects this approach. He stresses that there is no concept of scale in Iranian music. He goes on to say that the scale he mentions should be understood as a palette of all possible pitches used in Iranian classical music and as a source for creating modes. The modes created usually have four or five pitches and occasionally up to seven. He also reminds the reader that there is no chromatic movement in the melodies of classical Iranian music and that there are no intervals smaller than 90¢ in the structure of classical Iranian modes. This means that no melodic steps are used, e.g. between ♭E to 𝄳E or 𝄲F to 𝄳G; however, these smaller intervals are sometimes deployed as trills or ornaments and never used as main pitches within the context of the mode.

It should be noted that the results of **Dariush Safvat's** (1928–2013) **research**, published in *Musique d'Iran* (Caron and Safvat 1966), are almost identical to Farhat's research, except regarding the larger whole tone (220¢).

In his essay "A New Approach to the Theory of Persian Art Music: The Radif and the Modal System" (1993), **Dariush Talai** (1953) explains his personal procedure for analysing the Radif system, which is based on the assembling of four principal forms of tetrachords, or, as he calls them, "dâng", in various combinations to create the 12 *dastgah* of classical Iranian music and their *gusheh-ha*. These tetrachords are:



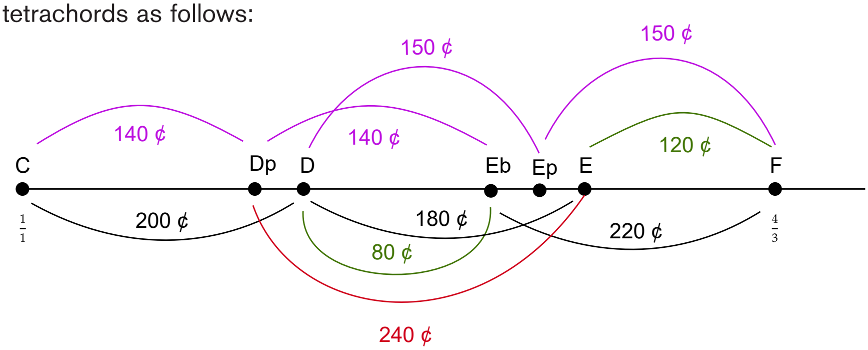
He adds that a single “dâng” is not enough to create music and explains the concept of the “do-dângi”, which is the combination of two adjacent “dâng”, where the last pitch in the first tetrachord is also the starting point for the following one:

♮G ♮A ♮B ♮**C + **♮**C** ♮D ♮E ♮F = ♮G ♮A ♮B ♮**C** ♮D ♮E ♮F

Together, these two tetrachords make a “do-dângi”, which is the potential gamut for the *dastgah-e* Mahur.

He explains that the intervals in the repertoire of Iranian classical music are flexible (oscillating between 80 and 270 cents), but this does not mean that this interval changes during the performance; instead it varies according to the type of tetrachord, the type of mode, the school and the personal style of the music. It is important to note that Talai clearly states that he bases his analysis of intervals on his own personal experiences and sensibilities (Talai, 2015, p. 22).

Based on his “Dâng” concept, Talai categorises the intervals used in the creation of the tetrachords as follows:



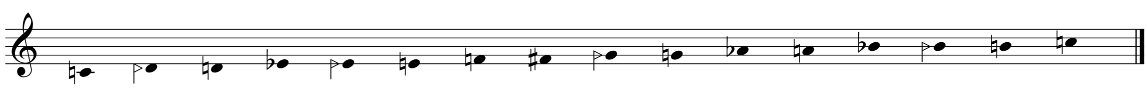
Seconds:

1- Minor = ½ tone ≈ 80¢ / 120¢
2- Neutral = ¾ tone ≈ 140¢ / 150¢
3- Major = 1 tone ≈ 180¢ / 200¢ / 220¢
4- Augmented = 5/4 tone ≈ 240¢

Thirds:

1- Minor = 1 & ½ tone ≈ 280¢
2- Neutral = 1 & ¾ tone ≈ 350¢
3- Major = 2 tones ≈ 380¢

And the potential chromatic gamut of classical Iranian music, or, as he mentions, the exact number and position of the frets and the sizes of the intervals on the contemporary tar instrument, is as follows:



The 15-tone scale of Talai

His theory of tetrachords and their combinations and permutations has its origins in the "Adwar System" from the 13th century, invented by Safiaddin Ormavi, the founder of the "Systematic Theory" and the way of thinking of music theory in Western Asia.

Another important international contribution to this subject is the study by **Jean During** (b. 1947), published in 1985 in the journal *Revue de Musicologie*. In his article "Iranian Scale Theories and Practices" he presented the measurement of intervals from the analysis of eight different recordings. As a result of his research, During found that there are two different sizes for the interval Mojannab (neutral second), as follows:

Mojannab type 1 (♮G to 𝄬A & ♮D to 𝄬E – 147¢)
Mojannab type 2 (𝄬A to ♭B – 139¢)

Interval	**minor second (m)**	**Mojannab 2 (n)**	**Mojannab 1 (N)**	**major second (M)**	**Plus tone**
Size in cents	90 (256/243)	139 (in general)	147 (in general)	204	–
	112 (in Rast)			182	

During presents all the intervals of the various *dastgah-ha* precisely and in detail. He claims that other researchers have somehow arbitrarily reduced the data and simplified the final results of their work. In any case, he uses the accidentals sori and koron in the same way in his transcription of the Radif as most theorists who present the general concept of the quarter tone of Vaziri.

Other Iranian theorists who write about the tuning and intonation of Iranian music are **Majid Kiani** (b. 1941) and **Siavash Beizai** (b. 1953).

To follow on from the work of Barkeshli and Savfat, both use mediaeval treatises on music as an analogy to determine exact interval sizes in classical Iranian music. Interestingly, both rely on the eleventh harmonic as a source of interpretation when they try to explain why a quarter tone is used in Iranian music.

Within his long article on the harmonic series, Beizai, in an attempt to justify quarter tones, clarifies that the melodic step 11:12 is responsible for the creation of the interval F sori to G in Iranian classical music and that the same harmonic could help us to develop other intervals, such as C to F sori (8:11) and neutral thirds D to F sori (9:11). The author goes on to say that all of these intervals can be tempered and concludes his article by presenting a 24-EDO chromatic scale for Iranian classical music.

In contrast to Beizai, who based his research on Vaziri's scale, mediaeval treatises and the harmonic series, Majid Kiani describes his research, which focuses on the analysis of the fret systems of Mirza Hossein-Qoli's tar (1853–1916) and the harmonic series, with an emphasis on the eleventh harmonic. Based on the ratios and scales presented by the mediaeval polymat hs Fārābi and Ormavi and on his research, Kiani presents the following ratios for the gamut on Mirza Hossein-Qoli's instrument and as possible intervals for classical Iranian music:

♮C	1/1	0
𝄳D	88/81	143.5
♮D	9/8	203.9
♭E	32/27	294.1
𝄳E	27/22	354.5
♮E	81/64	407.8
♮F	4/3	498.0
𝄲F	243/176	558.5
𝄳G	352/243	641.5
♮G	3/2	702.0
♭A	128/81	792.2
𝄳A	132/81	845.5
♮A	27/16	905.9
♭B	16/9	996.1
𝄳B	81/44	1056.5
♮C	2/1	1200

The 15-tone scale of Kiani

The following table summarises the intervals presented by the theorists mentioned in the first part of this article as components of a whole tone.

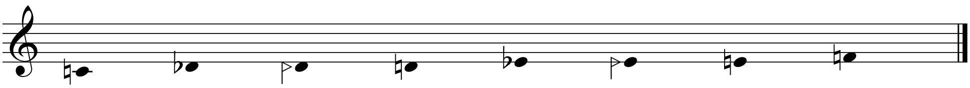
	Quarter tone	**Minor second (m)**	**lesser neutral second (n)**	**greater neutral second (N)**	**Major second (M)**	**Plus tone**
Vaziri	50	100	150	150	200	250
Barkeshli	–	90	120	180	204	–
Farhat	–	90	135	165	204	270
Talai	–	80–120	140	150	180–200–220	240
During	–	90–112	139	147	182–204	–
Beizai	–	112	151	156	182–204	236–248
Kiani	–	90	144	151	204	265

Division of the components of a whole tone based on the existing theories

On the notation of Iranian classical music

As can be seen from the preceding information, the size of intervals in Iranian classical music is variable. Consequently, the beauty and nuances of this music could be lost by simplifying the notation as a result of these intervals being tempered. Undoubtedly, all the theorists mentioned in this article have pointed out the same issue in their writings. Their aim was to capture these subtleties in their research. However, the lack of an efficient tool for the detailed notation of this music has caused them to overlook all of this precision and to notate the Radif system using the simplified notation system proposed by Vaziri a century earlier.

The following example shows the lower tetrachord of the potential gamut for creating Iranian modes with Vaziri's notation system:



This would be acceptable in the case of an equally tempered scale of quarter tones but is not sufficient to represent all the details in Iranian music.

As a composer who makes use of microtones in his music, I have tried various microtonal notation systems over the years to notate my music. Eventually, I found that the Extended Helmholtz-Ellis JI Pitch Notation (HEJI) is the most suitable notation system for my music and have been using it since 2015.

The HEJI system, conceived and developed by Marc Sabat (b. 1965) and Wolfgang von Schweinitz (b. 1953), enables the precise notation of all natural intervals. HEJI notation was developed for the composition and performance of new music using just intonation sonorities. It introduces new accidentals that raise and lower pitches by specified microtones. It also provides visually distinctive "logos" to distinguish families of natural intervals based on the harmonic series. As the authors of the system explain, HEJI notation enables the exact notation of all intervals that may be tuned directly by ear (natural intervals). It provides a method of writing any pitch height in the glissando continuum as a note on the five-line staff, and for each natural interval it indicates the harmonic relationships by which that note can be accurately tuned (Sabat 2005).

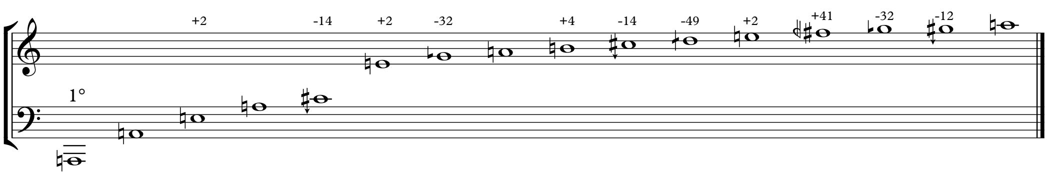
As can be seen in the table, the HEJI system has a new sign for the notation of each new prime partial within the harmonic series. Each partial that is a power of 2 and 3 belongs to the series of perfect fifths and fourths, known as Pythagorean tuning (3. limit), and is written using the "normal accidental" (♭♮♯). For each higher prime, there is an additional modifying accidental, which notates its deviation from a nearby Pythagorean note. There is an inverted version for each new sign for notating the subharmonic series.

Prime dimension	HEJI symbol	Symbol alters by
5	♭↓ ♮↓ ♯↓ | ♭↑ ♮↑ ♯↑	81/80
7	[illegible]	64/63
11	[illegible]	33/32
13	[illegible]	27/26
17	[illegible]	2187/2176
19	[illegible]	513/512
23	↓ | ↑	736/729
29	⇓ | ⇑	261/256
31	[illegible]	32/31
37	[illegible]	37/36
41	- | +	82/81
43	▾ | ▴	129/128
47	[illegible]	752/729

From *Plainsound Harmonic Space Calculator: Basic User Guide* **by Thomas Nicholson & Marc Sabat**

Here are the deviations from the tempered tuning system for each prime in the first few harmonic series:

Prime harmonic	Deviation from the equal temperament	HEJI accidental
3°	+2	♭♮♯
5°	-14	♭↓♮↓♯↓♭↑♮↑♯↑
7°	-31	[illegible]
11°	+51	[illegible]
13°	+41	[illegible]
17°	+5	[illegible]
19°	-2	[illegible]

Below is the notation of the first 16 harmonic partials of the A1 in HEJI system:

This notation system is easy to learn because it refers to the harmonic series. It is also important to know that the exact intonation of the intervals is easy to grasp, as they are all within the natural harmonics, which are easily found and produced on any instrument.

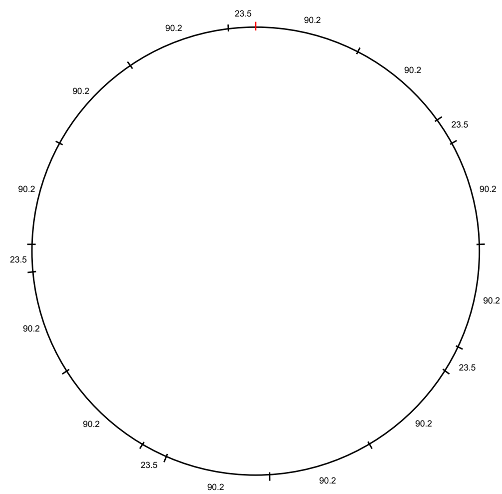
The 13th-Century Tuning System as a Starting Point

As a part of my doctoral research, I have found similarities between the size of the intervals introduced by the polymaths during this period and the intervals of contemporary classical Iranian music from studying the mediaeval treatises on music written in Iran between the 9th and 15th centuries. My research focuses on finding a solution for composing music with the modes and intervals of Iranian classical music using the compositional approach of rational intonation. I have found that one of the best approaches is to map the components of the modes onto the harmonic series.

The book *Kitab al-Adwar* by Safiaddin Ormavi, written around 1235 AD, is one of the most quoted books in the Middle East and also the starting point for my research. In short, all treatises on music in Iran before Ormavi are based on the description of the tuning systems for the existing instruments or *maqamat* (*maqamat* is the plural form for *maqam*) used in the region where the author of the book worked and lived. In his book, Ormavi introduces a system for describing the *maqamat* based on the combinations and permutations of the seven tetrachords and twelve pentachords. In a very detailed description of the intervals and the creations of *ajna*s (the plural form of *jin*s or tetrachord/pentachord), Ormavi describes 84 potential *maqamat* for the music of the Islamic world. Yet only 12 of them were known as the main *maqamat*.

Ormavi's system is based on the Pythagorean limmas and commas and is constructed using the circle of perfect fifths. The structure of the scale begins with $\{3^{-12}\}$ and goes up to $\{3^4\}$ as follows (Famourzadeh 2005):

$$\frac{1048576}{531441}, \frac{262144}{177147}, \frac{65536}{59049}, \frac{32768}{19683}, \frac{8192}{6561}, \frac{4096}{2187}, \frac{1024}{729}, \frac{256}{243}, \frac{128}{81}, \frac{32}{27}, \frac{16}{9}, \frac{4}{3}, \frac{1}{1}, \frac{3}{2}, \frac{9}{8}, \frac{27}{16}, \frac{81}{64}$$



The truth is that Ormavi was looking for a kind of temperament to devise a system to theorise about the music of his time, encompassing Persian, Arabic and Turkish music. Working with this theory in several of my compositions, I have found that the best and easiest way to use his modes is to map them into the harmonic series. The first and most important reason is that tuning to most of the ratios suggested by Ormavi in real time is impractical or even impossible. For example, the ear cannot tune a ratio such as $\frac{1048576}{531441}$, but within a rational intonation perspective, it is easier to tune the interval of $\frac{160}{81}$ (either as a natural harmonic or as a pitch) in the following example:

If $\frac{1}{1}$ is ♮E, then $\frac{1048576}{531441}$ is ♭F and $\frac{160}{81}$ is ♮↓E. The difference between ♭F and ♮↓E is lower than 2.0¢ or a Schisma.[1]

In the same case, ♮↓E with the ratio of $\frac{160}{81}$ with the ♮E as the reference pitch is equal to the $\frac{5}{4}$ from the ♮C, which could easily be found in the harmonic series of most orchestral instruments, for example on the ♮C string of the strings section or the fifth harmonic of the brass section. In that case, it is simple to have the ♮E as the reference tone, and to use ♮↓E instead of ♭F with the ratio of $\frac{1048576}{531441}$ in the harmonisation.

From the rational intonation perspective, I was able to find the alternate ratio for all the pitches with extremely high Pythagorean ratios in the lower harmonic series of orchestral instruments by looking at Ormavi's interval list:[2]

No	Phonetic transcription in the Abjad system	HEJI notation	Cents	Ratios proposed in *Kitāb al-Advār*	HEJI notation	The alternate ratio suggested by the author	Difference
1.	ا - a	♮E	0.00	1/1			
2.	ب - b	♮F	90.23	256/243			
3.	ج - j	♭G	180.45	65536/59049	♯↓F	10/9	2.0¢
4.	د - d	♯F	203.91	9/8			
5.	ه - h	♮G	294.13	32/27			
6.	و - v	♭A	384.36	8192/6561	♯↓G	5/4	2.0¢
7.	ز - z	♯G	407.82	81/64			

1 A schisma is the difference between a Pythagorean comma (531441/524288) and a syntonic comma (81/80) with the melodic ratio of 32768:32805.

2 Note that the objective is to find simpler ratios for each pitch in the gamut, so that they may be tuned to by ear or found in the lower harmonic series of instruments.

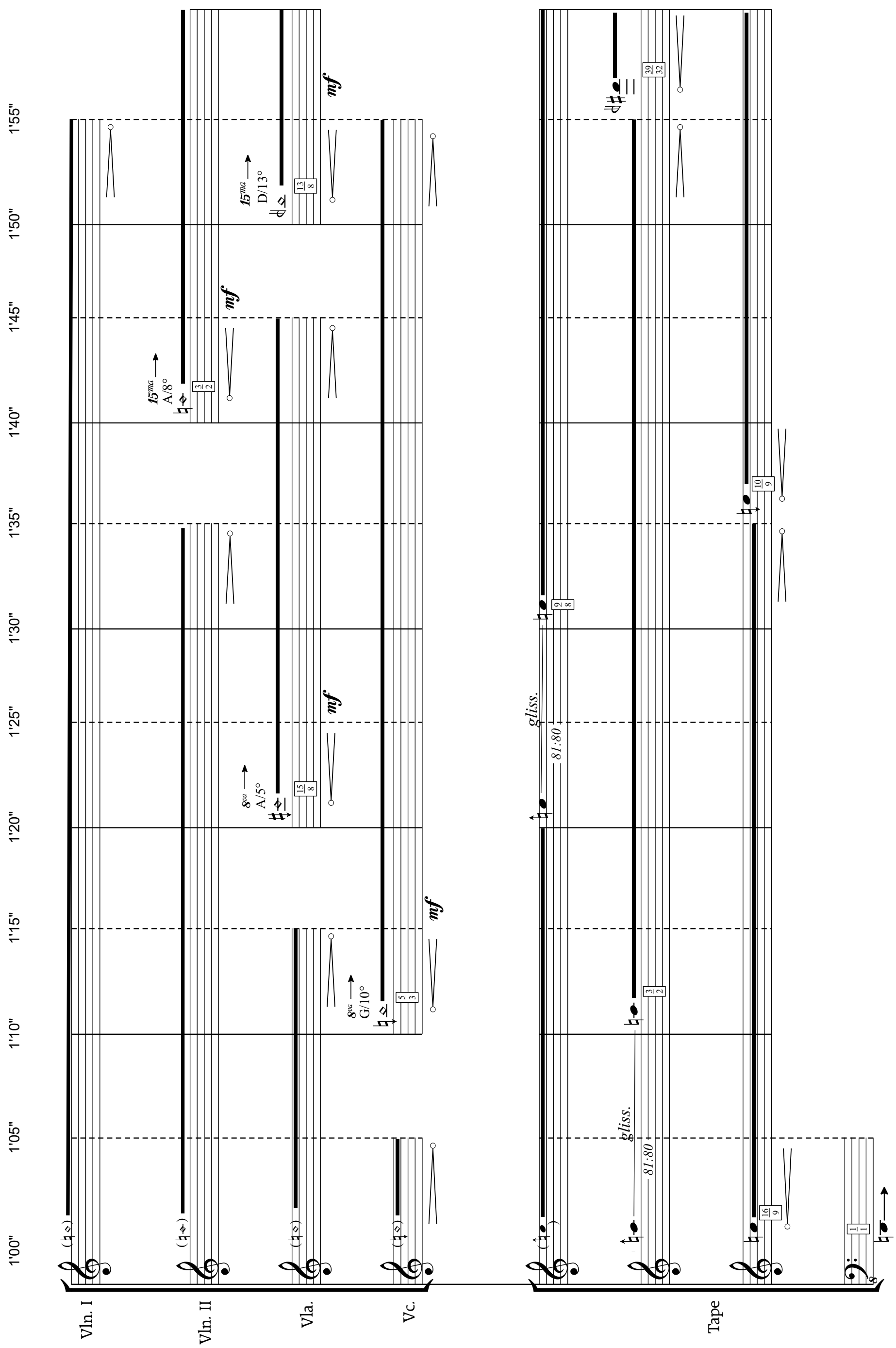
No	Phonetic transcription in the Abjad system	HEJI notation	Cents	Ratios proposed in *Kitāb al-Advār*	HEJI notation	The alternate ratio suggested by the author	Difference
8.	ح - ḥ	♮A	498.04	4/3			
9.	ط - ṭ	♭B	588.27	1024/729			
10.	ی - y	♭C	678.49	262144/177147	♮↓B	40/27	2.0¢
11.	ای - yā	♮B	701.96	3/2			
12.	یب - yeb	♮C	792.18	128/81			
13.	یج - yej	♭D	882.40	32768/19683	♯↓C	5/3	2.0¢
14.	ید - yed	♯C	905.87	27/16			
15.	یه - yeh	♮D	996.09	16/9			
16.	یو - yu	♭E	1086.31	4096/2187	♯↓D	15/8	2.0¢
17.	یز - yez	♭F	1176.54	1048576/531441	♮↓E	160/81	2.0¢
18.	یح - yeḥ	♮E	1200.00	2/1			

Ormavi's gamut compared with the rational intonation ratios

My composition *Crystallum* (2021), for string quartet and quadraphonic sound spatialisation, is an example of an actual composition with this idea applied. This piece is a journey into the depths of sounds and explores the inner and invisible relationships of their internal components and the complex network of their nature. The piece is structured using the interrelationship and mapping of all possible natural harmonic nodes, independently produced on the strings of each instrument of a string quartet. This construction is based on the highly complex interconnection of the overtones and subtones of a single pitch, ♮D, which for me plays the role of a tonal centre in relation to the open string spectrum of a string quartet, i.e., ♮C ♮G ♮D ♮A ♮E. This piece is a compositional kaleidoscope of networks organised with sounds in multiple layers and categories to be perceived from different angles.

In the example below, the tetrachord Rāst[3] [♮D ♮E ♯F ♮G] in the strings using the natural harmonics, confronted with the tetrachord No'ruz [♮A ♮↓B ♮C ♮D] in the electronics, leads through the modulation with glissandi to the tetrachord Arāq [♮A ♮↓B ♯↓C ♮D].

3 One of the seven tetrachords accepted by Ormavi. The following is a full list of the tetrachords he mentioned as the primary Ajnas for establishing Maqamat: **Ushshāq** [1/1 – 9/8 – 81/64 – 4/3], **Navā** [1/1 – 9/8 – 32/27 – 4/3], **Bousalik** [1/1 – 256/243 – 32/27 – 4/3], **Rāst** [1/1 – 9/8 – 8192/6561 – 4/3], **No`ruz** [1/1 – 65536/59049 – 32/27 – 4/3], **Arāq** [1/1 – 65536/59049 – 8192/6561 – 4/3], **Esfahān** [1/1 – 65536/59049 – 81/64 – 4/3].


1'00"
1'05"
1'10"
1'15"
1'20"
1'25"
1'30"
1'35"
1'40"
1'45"
1'50"
1'55"
Vln. I
Vln. II
Vla.
Vc.
Tape
8va G/10°
8va A/5°
15ma A/8°
15ma D/13°
mf
gliss.
81:80

Intervals of Iranian Classical Music in the Harmonic Series

While working on this idea and focusing on the tetrachord theories of Ormavi and Talai, I found that it is also possible to map the intervals of contemporary classical Iranian music into some higher harmonic series. I found that the relationship between the overtones 12 : 13 : 14 : 15 : 16 is identical to two of the most prominent tetrachords in Iranian music (if $\frac{1}{1}$ is ♮C then):

12 : 13 : 14 : 16 = 138 + 128 + 231 = ♮G 𝄳A ᒪ♭B ♮C = Shur
12 : 13 : 15 : 16 = 138 + 247 + 111 = ♮G 𝄳A ♮↓B ♮C = Chahargah

By considering the ♮G as the reference tone, we can write these tetrachords as:

$\frac{1}{1}$ $\frac{13}{12}$ $\frac{7}{6}$ $\frac{4}{3}$ ♮G 𝄳A ᒪ♭B ♮C = Shur

$\frac{1}{1}$ $\frac{13}{12}$ $\frac{5}{4}$ $\frac{4}{3}$ ♮G 𝄳A ♮↓B ♮C = Chahargah

On that basis, one can imagine that the relationship between the overtones 12 to 16 could serve as the basis for a viable proposition for the tuning system of classical Iranian music.

Harmonic series				
I (12:13:14:16)	1/1 ♮G	13/12 𝄳A	7/6 ᒪ♭B	4/3 ♮C
II (12:13:15:16)	1/1 ♮G	13/12 𝄳A	5/4 ♮↓B	4/3 ♮C
III (12:14:15:16)	1/1 ♮G	7/6 ᒪ♭B	5/4 ♮↓B	4/3 ♮C
IV* (13:14:15:16)	1/1 ♮G	14/13 ♯ᒪ♭A	15/13 ♯♮↓A	16/13 ♯♭B
V* (12:13:14:15)	1/1 ♮G	13/12 𝄳A	7/6 ᒪ♭B	5/4 ♮↓B

Different 4-note subsets between the harmonic series 12 to 16

Note that tetrachords I and II are the ones I have compared with the tetrachords Shur and Chahargah of classical Iranian music proposed by Talai, while tetrachords IV and V are incomplete. This means that they do not come to the perfect fourth. However, the sizes of the intervals between the notes are corroborated by the research of the scholars mentioned earlier.

It is exciting to note that tetrachord III (12 : 14 : 15 : 16) is the very tetrachord that Ibn Sinā (980–1037) had proposed as the famous chromatic *jins* of Persian music. In studying the works of Fārābi (9th century) and Ibn Sinā (10th century), I noticed their interest in higher prime numbers (i.e. those beyond 2, 3, 5 and sometimes 7), such as 13th and 11th. In their work, ratios such as $\frac{12}{11}$ or $\frac{13}{12}$ were used. In addition, in the following tables I will use the ratio $\frac{13}{11}$ as a minor third, a result of the combination of the two ratios mentioned in the works of the two great thinkers.

As a possible extension to the idea of finding the intervals of Persian music in the harmonic series, I have also imagined the entire fourth octave of the harmonic series as a potential palette for the construction of my scale:

Harmonic series	8	9	10	11	12	13	14	15	16
Ratio	1/1	9/8	5/4	11/8	3/2	13/8	7/4	15/8	2/1
HEJI	**♮C**	**♮D**	**♮↓E**	**𝄲F**	**♮G**	**𝄳A**	**ㄥ♭B**	**♮↓B**	**♮C**
from reference	0	203.9	386.3	551.3	702.0	840.5	968.8	1088.3	1200.0

Harmonic series 8 to 16 of ♮C

Consequently, with the help of the intervals I have found in the previous steps, I can place all four tetrachords proposed by Dariush Talai in the harmonic series:

Tetrachord	**Talai**	**Samimi Mofakham**
Mahur	♮C ♮D ♮E ♮F 200 + 180 + 120	♮C ♮D ♮↓E ♮F 204 +182 +112 $\frac{1}{1}$ $\frac{9}{8}$ $\frac{5}{4}$ $\frac{4}{3}$
Nava	♮C ♮D ♭E ♮F 200 + 80 + 220	♮C ♮D 𝄳𝄳E ♮F 204 + 85 + 209 $\frac{1}{1}$ $\frac{9}{8}$ $\frac{13}{11}$ $\frac{4}{3}$
Shur	♮C 𝄳D ♭E ♮F 140 + 140 + 220	♮C 𝄳D ㄥ♭E ♮F 138 + 128 + 231 $\frac{1}{1}$ $\frac{13}{12}$ $\frac{7}{6}$ $\frac{4}{3}$
Chahargah	♮C 𝄳D ♮E ♮F 140 + 240 + 120	♮C 𝄳D ♮↓E ♮F 138 + 247 + 112 $\frac{1}{1}$ $\frac{13}{12}$ $\frac{5}{4}$ $\frac{4}{3}$

Talai Tetrachords and their substitutes

Note that the original HEJI accidental for the ratio $\frac{13}{11}$ consists of two additional symbols for the 11th and the 13th harmonics (𝄳 and 𝄳), which concludes with a double accidental (in this case 𝄳𝄳E). As a recommendation from Marc Sabat, co-author of the HEJI notation system, to facilitate the reading of those ratios created based on the $\frac{13}{11}$ ratio, we proposed the following accidental: ♭ = 𝄳𝄳

The following is the list of the melodic distances between the components of the previous tetrachords, presented in HEJI as they were pitches from a reference tone:

Ratio	104:99	16:15	14:13	13:12	10:9	9:8	44:39	8:7	15:13
From ♮C	♭D	♭D	♯♭♭D	𝄳D	♮D	♮D	♯♯♭D	♭D	♯♮D
Size in cents	85.3	111.7	128.3	138.6	182.4	203.9	208.8	231.2	247.7

Component Intervals

By compiling all the pitches in the four different tetrachords and analysing the melodic steps between them, as below,

Ratio	1/1		104/99		13/12		9/8		7/6		13/11		5/4		4/3
	♮C		♭D		𝄳D		♮D		♭♭E		♭E		♮E		♮F
Size in cents	0		85.3		138.6		203.9		266.9		289.2		386.3		498.0
Melodic step		104/99		*32/33*		*26:27*		*27:28*		*77:78*		*52:55*		*15:16*	
melodic step in cents		85.3		*53.3*		*65.3*		*63.0*		*22.3*		*97.1*		*111.7*	

Division of the potential tetrachord

I would be able to propose the following structure as a gamut for creating different tetrachords of Iranian classical music:

No	Ratio	HEJI	Cents	Alternate simpler ratio	HEJI	difference
1.	1/1	♮C	0			
2.	104/99	♭D	85.3	21/20	♭♭D	0.8
3.	13/12	𝄳D	138.6			
4.	9/8	♮D	203.9			
5.	7/6	♭♭E	266.9			

No	Ratio	HEJI	Cents	Alternate simpler ratio	HEJI	difference
6.	13/11	♭E	289.2			
7.	5/4	♮E	386.3			
8.	4/3	♮F	498.0			
9.	416/297	♭G	583.3	7/5	♭G	0.8
10.	13/9	♭G	636.6			
11.	3/2	♮G	702.0			
12.	14/9	♭A	764.9			
13.	52/33	♭A	787.3	11/7	♮G	4.8
14.	5/3	♮A	884.4			
15.	16/9	♭B	996.1			
16.	1664/891	♭C	1081.4	28/15	♭C	0.8
17.	52/27	♭C	1134.7			
18.	2/1	♮C	1200.0			

Gamut I

Considering that this article's primary goal is to create a gamut with simple (lower) ratios, it is possible to use enharmonic[4] ratios for the following pitches (as indicated in the chart):

$\frac{21}{20}$ instead of $\frac{104}{99}$, $\frac{7}{5}$ instead of $\frac{416}{297}$ and $\frac{28}{15}$ instead of $\frac{1664}{891}$, all of them with 0.8 cents difference.[5]

And in the case of $\frac{52}{33}$ I have suggested $\frac{11}{7}$, which is the closest simple ratio and not exactly the enharmonic ratio for $\frac{52}{33}$, as the enharmonic ratio has an equally high ratio $\frac{63}{40}$ (♭A).

In the table Gamut I above, it is easy to observe that there are two great spaces between both $\frac{13}{11}$ and $\frac{5}{4}$ and $\frac{5}{4}$ to $\frac{4}{3}$, and the same pattern repeats itself between the ratios $\frac{52}{33}$, $\frac{5}{3}$ and $\frac{16}{9}$.

4 It is critical to remember that, in just intonation, there is no true enharmonic in the modern sense of a respelling of the same pitch. It is instead a re-understanding or reconceptualisation of the harmonic space through an almost unnoticeable shift in pitch.

5 Or Ibn Sinā's comma with the melodic ratio of 2080:2079.

One solution to smooth the steps between those intervals is to find ratios that could fit in between them. I have analysed all the possible epimoric ratios between the 8th and the 16th harmonic:

Ratio	9/8	10/9	11/10	12/11	13/12	14/13	15/14	16/15
HEJI	♮D	♮↓D	↑♭D	↓D	↓D	♯♭♭D	↑♯↓C	↑♭D
Size in cents	203.9	182.4	165.0	150.6	138.6	128.3	119.4	111.7

Melodic steps between the harmonic series (Epimoric $\frac{n+1}{n}$)

And all the epimeric ratios with two steps difference between the numerator and denominator:

Ratio	**9/7**	10/8	**11/9**	12/10	**13/11**	14/12	**15/13**	16/14
Normalised ratio		**5/4**		**6/5**		**7/6**		**8/7**
HEJI	↑E	♮↓E	↑♭E	↑♭E	↓↓E	♭♭E	♯♮↓D	↑D
Size in cents	435.1	386.3	347.4	315.6	289.2	266.9	247.7	231.2

Melodic steps between the harmonic series (Epimeric $\frac{n+a}{n}$ if a = 2)

For a smoother and broader range of palette, it is possible to add the aforementioned ratios to the gamut, including the Pythagorean and the rational ratios from Ormavi's gamut, including an interval of less than the semitone ($\frac{21}{20}$ and $\frac{256}{243}$), which I will suggest the ratio of $\frac{33}{32}$ (53.3 cents), that is the normalised result of the combination of the 11th and 12th harmonics:

$$\frac{11}{8} * \frac{12}{8} = \frac{33}{16} = \frac{33}{32}$$

By combining all of the ratios found in these various steps, the following gamut could serve as the most detailed order for the notation of Iranian classical music:

Ratio	Notation	Size from the reference	**Samimi Mofakham Gamut I**	**Samimi Mofakham additional ratios**	**Pythagorean/ Ormavi**
1/1	♮C	0	■		
33/32	↑C	53.3		■	
21/20	7♭↑D	84.5	■		
256/243	♭D	90.2			■
16/15	♭↑D	111.7		■	
15/14	ᒧ♯↓C	119.4		■	
14/13	13 7♭D	128.3		■	
13/12	13D	138.6	■		
12/11	↓D	150.6		■	
11/10	↑♭↑D	165.0		■	
10/9	♮↓D	182.4		■	
9/8	♮D	203.9	■		■
8/7	ᒧD	231.2		■	
15/13	13♮↓D	247.7		■	
7/6	7♭E	266.9	■		
13/11	13♭E	289.2	■		
32/27	♭E	294.1			■
6/5	♭↑E	315.6		■	
11/9	↑♭E	347.4		■	
16/13	13♭E	359.5		■	
5/4	♮↓E	386.3	■		
81/64	♮E	407.8			■
9/7	ᒧE	435.1		■	
4/3	♮F	498.0	■		■

Possible division of the tetrachord

Above is the proposed structure for the division of the lower tetrachords of the gamut. This can be used to notate the tiniest nuances of classical Iranian music and can be used to compose music with that can be read universally across different genres while maintaining these nuances. Ultimately, the research conducted for this article was not about implementing existing interval sizes on instruments but about proposing a range of intervals that could serve a purpose for practising musicians.

The Proposed Gamut Applied to an Actual Composition

Below is a concise example to show how the gamut might be used. It is a simple three-bar study for a brass quintet to explore the modulation between the three tetrachords of Mahur, Shur and Chahargah with supplementary passing tones, such as 33/32 and 256/243, to help smooth the melodic steps.

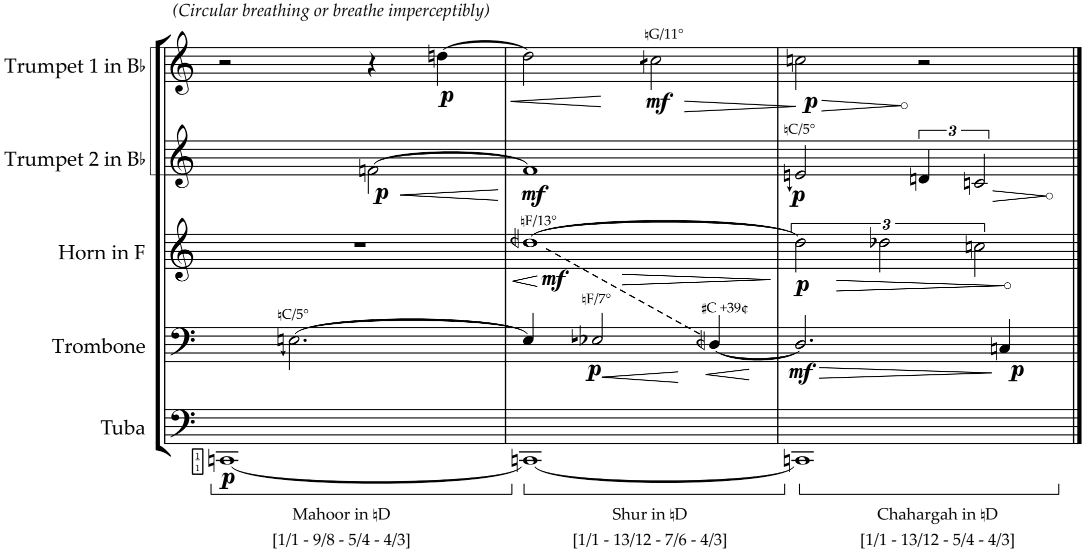
Study I

for Brass Quintet

as slow as possible

(Circular breathing or breathe imperceptibly)

Idin Samimi Mofakham



Score in C

Conclusion

Because of the differing views of Iranian music masters on the structure of the instruments and their tuning (whether through the analysis of older recordings or through analytical studies of the instruments), it is certainly not possible to provide an exact tuning for Iranian music. Researchers have thus far been content to provide a summary of intervals to precis their opinions. Their methods offered an average interval size and neglected the rich subtleties of intonation. Moreover, the scholars mentioned above present a scale for Iranian classical music without considering the nuances of intonation.

In my research, I want to show that it is necessary to look at this music differently. Instead of forcing the concept of the European scale onto Iranian music and building a scale, you can look at this music through the lens of harmonic space. This approach gives you a wide range of intervals to work with.

In conclusion, the rational intonation approach to Iranian classical music offers a new perspective useful to musicologists, composers and performers. By presenting this new perspective on the subject and offering an alternative approach to tuning and intonation, this research expands our understanding of Iranian classical music and provides valuable insights into the creative process of musical composition.

Whereas previous methods of understanding intervals in Iranian music have their shortcomings and oversimplifications, this new approach offers a more nuanced and holistic perspective.

References

Barkeshli, M. La gamme de la musique Iranienne. *Annales des Télécommunications*, 1950, 5(5), 195–203. https://doi.org/10.1007/bf03012055

Barkeshli, M. *"Ibn Sina and music" from on the Occasion of Celebrating Ibn Sina's Thousandth Birthday*. Iranian National Commission for UNESCO, 1981, 308–329.

Barkeshli, M. *Radif of Persian Classical Music, Collected by Musâ Ma'rufi*. Tehran: Mahoor Institute of Culture and Arts, 2011. (Original work published 1963.)

Beizai, S. The Origins of the Quarter-Tone in Persian and Arabian Music. *Honar Quarterly*, 2002, (51), 114–134.

Binesh, T. *Three Persian Treatises on Music*. Tehran: Nashre Daneshgahi Publications, 1992.

Blum, S., & Farhat, H. Reviewed Work: The Dastgāh Concept in Persian Music by Hormoz Farhat. *Ethnomusicology*, 1992, 36(3), 422. https://doi.org/10.2307/851875

Caron, N., & Safvat, D. *Musique d'Iran*. Paris: Buchet-Chastel, 1997 (Original work published 1966), 25–39.

Danielson, V., Reynolds, D. and Marcus, S. (eds.). *The Middle East (Garland Encyclopedia of World Music, Volume 6)*. New York: Routledge, 2002, XXVII, 1182.

During, J. Théories et pratiques de la gamme iranienne. *Revue de Musicologie*, 1985, 71(1/2), 79. https://doi.org/10.2307/928594

During, J. *The Radif of Mirza Abdollah. Interpreted by Nur'ali Borumand. A Canonic Repertoire of Persian Music* Tehran: Mahoor Institute of Culture and Art, 2014, 56.

Famourzadeh, V. *La musique persane, formalisation algébrique des structures.* Master's thesis, Université du Maine, 2005. Retrieved from http://recherche.ircam.fr/equipes/repmus/mamux/documents/VedadMasterThesis.pdf

Farhat, H. *The Dastgāh Concept in Persian Music*. Cambridge: Cambridge University Press, 2004.

Farmer, H. G. The Lute Scale of Avicenna. *Journal of the Royal Asiatic Society of Great Britain & Ireland*, 1937, 69(2), 245–253. https://doi.org/10.1017/s0035869x00085397

Forster, C. *Musical Mathematics: On the Art and Science of Acoustic Instruments*. San Francisco: Chronicle Books, 2010.

Khaleghi, R. *A Glance at the Theory of [Persian] Music*. Tehran: Safi Ali-Shah Publication, 1938.

Kiani, M. *Seven Dastgah(s) of Iranian Music*. Teheran: Soore Mehr Publication, 1992.

Nicholson, T., & Sabat, M. *Fundamental Principles of Just Intonation and Microtonal Composition* (2018, ongoing).

Nicholson, T., & Sabat, M. Farey Sequences Map Playable Nodes on a String. *Tempo*, 2019, 74(291), 86–97. https://doi.org/10.1017/s0040298219001001

Sabat, Marc. The Helmholtz Ellis JI Pitch Notation: Microtonal accidentals designed by / mikrotonale Vorzeichen konzipiert von Marc Sabat & Wolfgang von Schweinitz, 2004-2005 Plainsound Music Edition

Tala'I, D. نگرشي نو به تئوري موسيقي ايراني (رديف و سيستم مدال)/ *A New Approach to the Theory of Persian Art Music*. Tehran: Mahoor Institute of Culture and Arts, 1993.

Tala'I, D. *Traditional Persian Art Music: The Radif of Mirza Abdullah*. New York: Bibliotheca Persica, 1999.

Talai, D. *Radif Analysis Based on the Notation of Mirza Abdollah's Radif with Annotated Visual Description*. Tehran: Nashr-e Ney, 2015.

Vaziri, A.-N. *The Harmony of the Music of Iran or Quarter-tone Music*. Tehran: Mahoor Institute of Culture and Art, 2016. (Original work published 1935.)

Yousef, Z. ya, El Hefni, M. A., & El-Ahwani, A. F. *Al-Shifa. Mathematiques. 3.-Musique (jauami' 'ilm el-musiq)/ al-shifa: al-Riyadiyat.* Cairo: Organisme Général des Imprimeries Gouvernementales, 1956.

The music of Iranian composer/performer **Idin Samimi Mofakham** (*1982) is strongly influenced by the traditional and regional music of his home country. Since 2015, he has developed his own musical language based on mediaeval Persian tuning systems, just intonation and psychoacoustics. He completed his PhD at the Norwegian Academy of Music in Oslo, Norway, where he researched Persian mediaeval tuning systems and their creative use in contemporary composition.