A Chord Quality Taxonomy from Karel Janeček's *Základy moderní harmonie* (*Foundations of Modern Harmony*, 1965) and an American set-theoretical interpretation

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Abstract: Karel Janeček's *Základy moderní harmonie* (1965), while mostly unknown outside of Czech-speaking theoretical circles, addresses a variety of topics relevant to current theoretical discussions of twentieth century music, including but not limited to, chord quality, harmonic function, and prolongation. Besides remaining yet untranslated, the text provides challenges in complexity, idiosyncrasy, and scope. This study translates selected excerpts from *Základy moderní harmonie* to explain Janeček's theory of chord quality taxonomy and then interprets it in traditional set theoretical language after theorists like Allen Forte, Robert Morris, and John Rahn. Two findings arise from my interpretation of Janeček's taxonomy. The first is historical: Janeček's approach is one of the earliest predecessors to the set-theoretical method initiated by Allen Forte. The second is theoretical with analytical implications: Janeček recognizes that different intervals have variable affective influence on chord quality. I also discuss Janeček's implicit engagement with other contemporary theorists like Alois Hába, Paul Hindemith, and Arnold Schoenberg.

Keywords: Czech music theory, Janeček, set-theory, history of theory, chord quality

Karel Janeček's *Základy moderní harmonie*, while mostly unknown outside of Czech-speaking theoretical circles, addresses a variety of topics relevant to current theoretical discussions of twentieth-century music, including, but not limited to, chord quality, harmonic function, and prolongation. Besides remaining yet untranslated, the text provides challenges in complexity, idiosyncrasy, and scope. This study translates selected excerpts from *Základy moderní harmonie* to explain Janeček's theory of chord quality taxonomy and then interprets it in traditional set-theoretical language after theorists like Allen Forte, Robert Morris, and John Rahn. I focus on Janeček's taxonomy, as opposed to his other theoretical topics, for two reasons: first, the pervasiveness of theories of chord quality before and after Janeček facilitates comparing him to other authors and placing him historically; second, his taxonomy is pivotal to understanding his equally thorough and intriguing theoretical work on harmonic function and prolongation in contemporary music, also addressed in *Základy*.

The inspiration for this project is not necessarily limited to history. With music-theoretical research on issues in twentieth-century music still blossoming, older theories can serve as seed for further developments. The work of Daniel Harrison (2016) and Ian Quinn (2004) serve as prime examples. In developing his original ideas, Harrison invokes statements made by Paul Hindemith (1937) and Ernst Krenek (1940) to support the notion that the effect of a chord resides in its realization through voicing. Harrison then continues to borrow from Hindemith to establish *Hindemith-lines*, a type of step progression based on Hindemith's ideas, with important differences from Schenkerian linear progressions. For Quinn, Howard Hanson's (1960) work forms the basis of a generic taxonomy.

Two findings arise from my interpretation of Janeček's taxonomy. The first is historical: Janeček's approach is one of the earliest predecessors to the set-theoretical method initiated by Allen Forte. The second is theoretical with analytical implications: Janeček recognizes that different intervals have variable affective influence on chord quality. In his taxonomy, the rules of genus membership, based on the inclusion of particular subsets, prove to be novel. I first provide the necessary translated excerpts from *Základy* to understand Janeček's taxonomy, while highlighting the similarities and differences of his approach with other theoretical approaches. I then provide my interpretation and an explanation of it. Where appropriate throughout the study, I highlight Janeček's implicit inspirations or references to other authors, like Alois Hába, Hindemith, and Arnold Schoenberg. I conclude by speculating on the potential relevance of Janeček's taxonomy today.

The text in question, Janeček's *Základy moderní harmonie*, was published while he was a professor at the Academy of Performing Arts (*Adakemie múzických umění*) in Prague. Intended as an aid to contemporary composition, *Základy* addresses more than just the chord quality taxonomy that is the topic of this study. It also provides an alternative taxonomic approach, an approach to harmonic function for all possible sonorities, identification of non-harmonic tones in modern music, and approaches to composing and analyzing music in the modernist style. While for Janeček music theory was second to composition, he considered theoretical and analytical work a pivotal part of composition. In an earlier article, Janeček makes his views on the relationship between theory and composition explicit:

> Such coexisting contradictions as theory and production reveal the fundamental, uniquely unidirectional relationships between them: theory aids production, serves it. In fact, this is its primary aim, and it is also its permanent fundamental mission. If this function of theory is to be discarded, meaning is lost; it is no longer needed.[1] (Janeček 1957, 920)

Given these views, it is perhaps not surprising that his approach is both intuitive and systematic, as will become clear through this study. He attempts to build from a groundwork of experience and observation as a composer to gain insights into the musical possibilities of the modernist era, of which no one yet has experience, and he does so with systematic theoretical study.

Janeček explains that the inherent task of *Základy* is "the consideration of the possible organization of harmonic materials," which "is exhaustible" (1965, 12).[2] Such a task is not necessarily novel. Ernst Bacon (1917) appears to be the first author to provide an exhaustive list of possible harmonies. Both Bacon's list and Janeček's include the 350 possible sonorities when assuming octave, enharmonic, and transpositional equivalence. While Janeček does not use these exact terms, the following statements make his stance clear:

> On enharmonic equivalence – "If we designate the half-tone as the smallest possible interval 1, 2 will represent the whole tone, 3 the minor third, 4 the major third, 5 the perfect fourth, 6 the augmented fourth or tritone, 7 the perfect fifth etc."[3] (1965, 21)

> On transpositional equivalence – "Two sonorities are of the same type not only if they entirely coincide in all parts, but also if it is possible to convert them to the same formation through reordering and transposition."[4] (1965, 22)

1 Takové soužití protikladů, jakými jsou teorie a tvorba, prozrazuje základní, jednoznačně jednosměrný vztah mezi nimi: *teorie pomáhá tvorbě, slouží jí.* V tom je vskutku její prvotné poslání, v tom je i její *trvalé poslání základní*. Zbaví-li se teorie této funkce, pozbývá smyslu; není k potřebě.

2 Vlastní práce na *Základech* začala úvahami o možnosti třídění harmonického materiálu. Charakterové roztřídění souzvuků lze propracovat beze zbytku.

3 Označíme-li půltón jakožto nejmenší možný interval 1, bude znamenat 2 celý tón, 3 malou tercii, 5 velkou tercii, 5 čistou kvartu, 6 zvětšenou kvartu čili tritonus, 7 čistou kvintu atd.

4 Dva souzvuky jsou téhož druhu nejen tehdy, shodují-li se ve všem (do všech podrobností) jako jednotliviny, nýbrž i tehdy, *dají-li se převést na shodné útvary* úpravou a transpozicí.

> On octave equivalence – "We consider transfer by an octave not to be transposition, but repetition."[5] (1965, 21)

Any sonorities related by reordering, enharmonic respelling, or transposition are effectively considered equivalent, leaving 350 unique sonorities unrelated by reordering, respelling, or transposition. The reordering of, or permutation of, a sonority relates to the concept of chord inversion in traditional harmony; both authors recognize that two sonorities that are related to one another by reordering the pitches of the sonority or through permutation are essentially the same sonority. Notably, neither author assumes the inversional equivalence of sonorities in their lists – inversion in the sense that the intervallic inversion of a major chord is a minor chord and the intervallic inversion of a minor chord is a major chord.

Given the obscurity of Bacon's work, the geographical and chronological isolation of the two authors, and the omission of Bacon from the bibliography for *Základy*, it is unlikely that Janeček was aware of Bacon's article. Janeček was, on the other hand, undoubtedly aware of the work of Czech composer and theorist Alois Hába, given that he cites Hába in the bibliography for *Základy*. Hába (1927) takes on the task of enumerating as many harmonies as possible in his *Neue Harmonielehre*. Hába's approach is highly systematic, taking an interval and adding to it in every possible manner in every position. While Hába's attempt is neither complete, tidy, nor without redundancies, the systematic portion of Janeček's work, if not directly inspired by Hába, is certainly reminiscent of it.

Towards his goal of categorizing all possible harmonic materials, Janeček proposes naming sonorities according to their *orientation scheme* (*orientační schéma*), rather than organizing them into triads to determine their type:

> In classical harmony, we simply arrange the tones of the chord in question into a triad, so that we can recognize its type. With sonorities (*souzvuky*) that arise in modern music as with dissonant sonorities [continuous harmonies (*průběžný harmonie*)] in classical music, this practical method cannot be used in most cases; in most cases the tones of a sonority cannot be arranged into triads. Therefore, we must agree on another suitable method that can be applied consistently to all possible sonorities in equal temperament. (1965, 22)[6]

[5] Přenesení o oktávu nepovažujeme za transpozici, nýbrž za opakování.

[6] V klasické harmonii prostě uspořádáme tóny posuzovaného akordu to tercií, abychom poznali jeho druh. U souzvuků, jež se vyskytují v moderní hudbě, a u průběžných harmonií v hudbě klasické, lze tohoto praktického způsobu použít jen ve zvláštních případech; ve většině případů tóny souzvuku do tercií srovnat nelze. Musíme se tudíž dohodnout na jiném vhodném způsobu, jejž by bylo možno uplatnit *důsledně u všech vůbec možných souzvuků v temperované chromatice*.

In this quotation, we find Janeček borrowing, without citation, from Hába (1927), who proposes that constructing chords in thirds or fourths is unnecessary and that all intervals should be admitted. Janeček's statement is likely also an implicit reference to Hindemith (1937), another source cited in Janeček's bibliography, who famously also created a taxonomy of chord quality. Hindemith, however, did organize chords into triads. As will be clear later, the value of Janeček's approach is that he is not, like Hindemith, left with chords of indeterminate roots lumped together into their own genus. Janeček's rejection of Hindemith's approach will prove more salient given the other similarities between their taxonomies, to be explored later.

In place of an explicit definition of his *orientation scheme*, Janeček provides the following two instructions for finding it:

1. The pitches of the sonority are arranged to have the smallest range possible.
2. Should it occur that there are several arrangements with the same range, then we select the one in which the intervals from bottom to top are arranged from smallest to largest. [7] (1965, 22)

We can quickly recognize that the process described by Janeček is essentially identical to the process for finding normal order. The normal order of a sonority is a reduction of a sonority to a basic ordered pattern (Forte 1973). Normal order looks past differences in ordering or permutation in a sonority and is considered unordered. Forte's (1973) procedure for finding normal order is provided here for comparison:

> "Requirement 1, the normal order is that permutation with the least difference determined by subtracting the first integer from the last."
>
> "Requirement 2 selects the best normal order as follows. If the least difference of first and last integers is the same for any two permutations, select the permutation with the least difference between first and second integers. If this is the same, select the permutation with the least difference between the first and third integers, and so on, until the difference between the first and the next to last integers has been checked. If the differences are the same each time, select one ordering arbitrarily as the normal order." (1973, 4)

Forte's *requirement 1* and Janeček's first step are essentially identical, while Forte's *requirement 2* is a detailed account of how to achieve the arrangement of intervals from smallest to largest as described in Janeček's second step.

[7] 1. Tóny souzvuku se uspořádají do *nejmenšího možného rozsahu*. 2. Vychází-li několik tvarů o stejném nejmenším rozsahu, volíme ten, v němž interval jsou zdola nahoru uspořádány *od menších k větším*.

Janeček provides the following example: "The orientation scheme of the sonority e g c d would be c d e g, because the other orderings have a larger range (e–d, g–e, d–c). The orientation scheme of the sonority g c d would be c d g, because in contrast from g c d, which has the same span, here the intervals are arranged from smallest to largest" (1965, 22).[8] Figure 1 provides visual clarification of Janeček's explanation. When Janeček provides pitch names he is implicitly referring to pitch-classes rather than pitches. The letter e abstractly represents e in any octave or enharmonic spelling, as opposed to a pitch, which in set theory traditionally implies a pitch in a specific octave. Further, when these pitch-classes are grouped together, they form different pitch-class sets, which Forte (1973) defines as "a set of distinct integers representing pitch classes … often abbreviated to pc set." Moving forward, I will adopt the set-theoretical term *pc set* to replace Janeček's *sonority* (*souzvuk*).

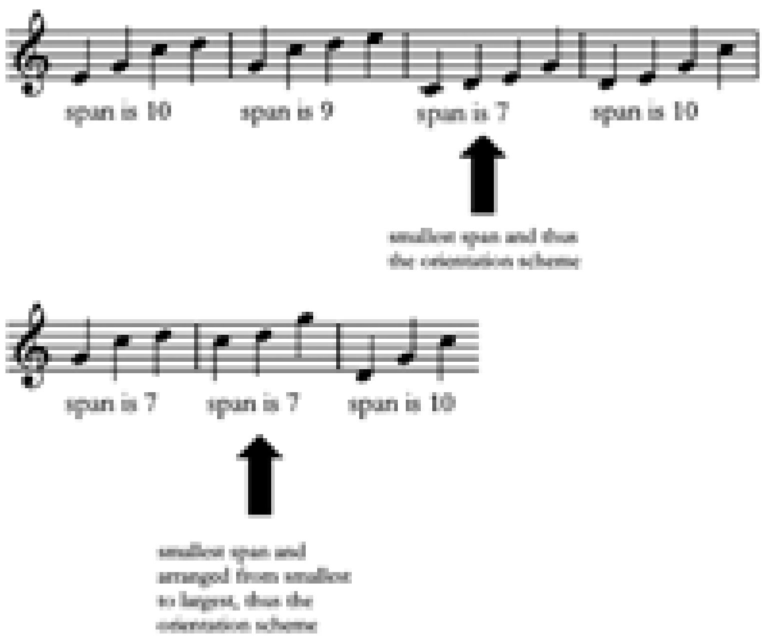

Figure 1. A visual representation of Janeček's textual orientation scheme example.

[8] Orientační schéma souzvuku *e g c d* bude tedy *c d e g*, neboť ostatní tvary mají rozsahy větší (*e–d, g–e, d–c*). Orientační schéma souzvuku *g c d* bude *c d g*, neboť na rozdíl od daného tvaru *g c d*, který má stejný rozsah, jsou zde intervaly uspořádány od menšího k většímu.

The orientation scheme of a pc set is an abstraction that could represent any ordering of the pitch-classes in the orientation scheme. The orientation scheme of a pc set is thus unordered, just like the normal order of a pc set. The difference between normal order and an orientation scheme, however, is that Janeček does not use either letter or integer notation to represent the pitch-classes of orientation schemes. Instead, he uses integers to label the "neighboring intervals from bottom to top" (1965, 22).[9] He provides the example c d e g as orientation scheme 223, as shown in Figure 2.

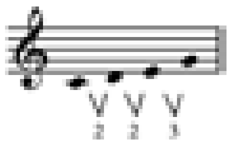

Figure 2. The pc set c d e g with neighboring intervals labeled to show its orientation scheme – 223.

We find a similar decision in Chrisman (1969), where pc sets are represented by their *successive-interval array* – the "successive absolute differences between adjacent pitch-class members." Morris's (1988) *INT* and *CINT* similarly enumerate the intervals between pitch-classes. Focusing on Janeček, the effect of this decision is that his orientation scheme is a literal representation that can simultaneously represent all possible transpositions of a given pc set, thus observing transpositional equivalence. Figure 3 provides an example of this. The intervals between pitch-classes, as represented by his orientation scheme, are literally included in every transposition of that pc set. On the other hand, to observe transpositional equivalence when pitches are represented with some notation other than intervals requires the additional step of saying that one transposition of that pc set will act as an abstract representation of all possible transpositions.

Figure 3. The twelve possible transpositions of a pc set represented by the orientation scheme 223.

[9] *Obecně* označujeme orientační schéma souzvuku *číslicemi* podle *sousedících intervalů* zdola nahoru.

Given Janeček's explanation of his orientation scheme, we can define it with set-theoretical language: the orientation scheme of a pc set is its T_n *type*, represented with a successive interval array (SIA). T_n *types* are all unordered pc sets after observing enharmonic, octave, and transpositional equivalence with a twelve-note division of the octave, as opposed to *Tn/TnI types*, which observe inversional equivalence. Both terms were introduced by John Rahn (1980). We can also say that T_n *types* are also essentially all pc sets in normal order transposed to begin on zero to account for transpositional equivalence. Janeček's process for finding the orientation scheme is essentially equivalent to any approach for finding normal order that observes the preference rule for pc sets more closely packed to the left. An SIA is simply a representation of the intervals between pitches or pitch-classes in a pc set, like in Figure 2. As previously mentioned, an SIA literally represents all twelve transpositions of a pc set. Because of this, when Janeček takes pc sets that we would say are in normal order and labels them with an SIA, he essentially applies transpositional equivalence to normal order, which is the process by which to find T_n *types*. Given that there is a one-to-one relationship between any SIA and the T_n *type* it represents, I will from here on abandon the term orientation scheme for T_n *type* and simply represent it as a pc set rather than an SIA. The key aspect of this as it relates to my summary of Janeček's taxonomic approach is that he does not recognize inversional equivalence and thus is working with 350 pc sets to categorize.

Now I will explain the taxonomic decisions in Janeček's approach. The primary assumption governing Janeček's approach to chord quality taxonomy is his view on consonance and dissonance. First, he defines the two terms as follows: a consonance is "that sort of sonority that can in favorable surroundings influence the listener in such a way that in him a tension is not raised," and a dissonance is "such a sonority that inspires in the listener a feeling of unrest and tension in all surroundings, i.e., in any configuration and in any position of an order of sound" (1965, 46–47).[10] His emphasis on favorable surroundings refers essentially to harmonic progression. He recognizes something as a consonance if there is any harmonic progression in which that sonority does not engender tension within the listener. This allows for situations like a dominant triad, which he recognizes as creating tension not due to an inherent dissonance, but rather to its relationship to the other chords in the progression (1965, 207). For Janeček, the major triad is still a consonance under his definition even though it does create tension under certain situations. A dissonance, on the other hand, will always create tension regardless of its place in a harmonic progression.

Janeček explicated his primary assumption about consonance and dissonance as follows:

[10] *Konsonance je takový souzvukový druh, který může za příznivých okolností působit na posluchače tak, že v něm nevzbuzuje napětí. Disonance je takový souzvukový druh, který vzbuzuje v posluchači pocit neklidu a napětí za všech okolností, tj. v jakékoliv úpravě a v jakémkoliv postavení v souzvukovém sledu.*

> I consider the prevalent view about the shifting of the boundary between consonances and dissonances to be [a rooted preconception passed from book to book]. In almost every interpretation of modern harmony, I have encountered this view (and even supported it myself). I finally realized that with such a bias it is not possible to honestly solve the problems of harmony. It is precisely that rich diversity of existing dissonances that tempts theorists to think that chords once considered dissonant have now become consonant and that the range of consonances is thus expanding. Although I have studied dissonances thoroughly, never have I converted any of them to consonances. I realized that shifting the boundary between consonances and dissonances meant admitting that there is no substantial difference in sound between these two categories of chords. The superiority bestowed on consonances by nature would thus be mere fiction. (1965, 11)[11]

Though not made explicit, this is likely a reference to Schoenberg's famous comments on the evolution of consonance and dissonance from his *Harmonielehre*, which Janeček also included in the bibliography to *Základy*. Given his view on consonance and dissonance, Janeček proposes two different approaches to chord classification: one based on dissonance and one based on consonance. It is solely the classification based on dissonance that is the topic of this study.

According to Janeček, "Dissonances are highly varied. We find among them sonorities that are sweet, pleasantly irritating, faintly clouded, sharply focused, irritatingly harsh, translucently or chaotically overloaded" (1965, 47).[12] He explains that "the large variety of possible dissonances result in the necessity to order them" (1965, 47).[13] To this end he recognizes four dissonant elements (*disonantní prvky*). These dissonant elements are components (*složky*), which he describes finding as follows:

> If we proceed from complex-sounding dissonant sonorities of higher orders to simpler-sounding sonorities of lower through the process of gradually removing individual tones, and with it also individual intervallic components, we will ultimately always come to a formation that is sonically transparent, simple, consonant: to a trichord, a dyad, or a monad. Therefore, there exist limits, at the crossing of which sonic complexity and tension (dissonance) disappear. It apparently

[11] Za takový předsudek považuji např. velmi rozšířený názor o posuvnosti hranice mezi konsonancemi a disonancemi. Téměř v každém výkladu o moderní harmonice jsem se s tímto názorem setkal (a sám jsem jej těž dříve zastával). Uvědomil jsem si nakonec, že s takovým zatížením nelze vlastně vůbec otázky harmonie poctivě řešit. Je to právě bohatství a rozmanitost disonancí, co svádí teoretiky k názoru, že souzvuky kdysi disonantní se staly v nové době konsonantními, a že se tudíž okruh konsonancí rozšiřuje. Třebaže jsem se zabýval studiem disonancí velmi důkladně, přece jsem žádnou z nich nepřevedl mezi konsonance. Byl jsem si vědom toho, že posouvání hranice mezi konsonancemi a disonancemi znamená vlastně přiznání, že mezi oběma těmito souzvukovými kategoriemi není podstatného zvukového rozdílu, a že tudíž i přírodou daná nadřaděnost konsonancí je pouhá fikce.

[12] Disonance jsou *velmi rozmanité*. Najdeme mezi nimi souzvuky líbezné, příjemně dráždivé, mdle zakalené, břitce zaostřené a drásavě drsné, průzračné i chaoticky přetížené.

[13] Z velkého množství možných disonancí vyplývá nutnost jejich *třídění*.

happens because components of the sonority fall away: components that cause sonic distortions or dissonance. (1965, 47)[14]

Essentially, Janeček describes a process whereby dissonances are identified by removing notes and thus intervals from more complex pc sets until the pc sets become consonant. The final components removed before the pc set becomes consonant must be the contributors to the previously heard dissonance. These components, these dissonant elements, he identifies as the semitone (01), the whole tone (02), the tritone (06), and the augmented triad (048). Given his statement that "the inversion of a sonority is just as consonant or has the same measure of dissonance as the original sonority," it would be appropriate in current language to say that he invokes interval classes (ic) – the idea that any intervals, after inversion or octave transposition, can be reduced down to one of seven basic intervals from null to six (1965, 49).[15] However, because these dissonant elements include pc sets, not just intervals, I will represent these elements visually as pc sets instead of intervals. For Janeček, each of these dissonant elements has a unique sonic quality (*charakteristika*) and they are what define the *dissonant characteristic* (*charakteristika*) of a sonority.[16] These are the intuitive assumptions, that I mentioned earlier, that Janeček then systematically builds on, as we will now see.

For example, all sonorities, in which out of those four dissonant elements only (01) is included – i.e., in which out of those four dissonant elements only the minor second is included – have the same dissonant characteristic. This dissonant characteristic is the inclusion of (01) and the exclusion of all other dissonant elements. Thus, we can say that they constitute a single genus with the dissonant characteristic (01). Figure 4 shows all members of this genus. The same applies to sonorities that contain only dissonant elements (02), (06), or (048):

Genus (01) members

(01)			
(014)	(045)	(015)	(034)
(0347)	(0158)	(0145)	

Figure 4. All members of the genus including only (01). Each member of this genus includes only the pc set (01) from Janeček's list of dissonant elements.

14 Postupujeme-li zvukově složitých disonantních souzvuků vyšších tříd ke zvukově jednodušším souzvukům tříd nižších tím způsobem, že odstraňujeme postupně jednotlivé tóny, a tím také jednotlivé intervalové složky, dojdeme nakonec vždy k útvarům zvukově průzračným, jenoduchým, *konsonantním*: k trojzvukům, dvojzvukům nebo k jednozvuku. Existují tedy určité meze, při jejichž překročení zvuková složitost a napětí (*disonantnost*) mizí. Stane se tak zřejmě tím, že ze souzvuku odpadnou složky, které zvukové zauzlení či *disonantnost* způsobují.

15 *Inverze souzvuky je stejně konsonantní nebo má stejnou míru disonantnosti jako souzvuk původní.*

16 I have used the term *quality* in reference to the sound of each individual dissonance instead of *characteristic* to avoid overlap with the term *dissonant characteristic*.

Earlier, I mentioned that similarities exist between Janeček's and Hindemith's approaches. Janeček's view of the semitone, the whole tone, the tritone, and the augmented triad as distinctive factors of chord quality agrees with Hindemith (1937), who uses the same dissonances to categorize chords in his own taxonomy. Given Janeček's inclusion of Hindemith (1937) in the bibliography, it seems undoubtable that Janeček was aware of Hindemith's approach and that Janeček may have drawn inspiration from him. Janeček neither confirms or denies this in *Základy*. As we will see, though, Janeček greatly expands on these preliminary four dissonances he shares with Hindemith and thus makes his taxonomy more defined.

Janeček allows for any combination of those dissonant elements to serve also as a dissonant characteristic. Thus, his list of dissonant characteristics grows to include the combinations of sonorities in Figure 5. He also makes a special case for two other dissonant elements: (012) and (0167). He explains that these two situations do not represent merely an increase in, or a combination of, dissonant elements. Rather, with these two sonorities we encounter a "change in quality" (1965, 61).[17] He explains that "The use of the semitone elements, which are a semitone apart [(012)], and the use of two tritone elements, which are a semitone apart [(0167)], represent in fact the intervention of a new factor" (1965, 61).[18] Figure 6 shows the sets in question. The point is that, for Janeček, (012) and (0167) represent two dissonances so distinct from their constituent dissonant elements that they represent dissonant elements themselves. We can see a similarity between these sets and (048) as dissonant elements. For the augmented triad, three instances of a consonance (04) in a particular configuration create a dissonance. For (012) and (0167), the particular ordering of the dissonances (01) and (06) respectively constitute new dissonant elements. Including these two new elements thus expands the list of dissonant characteristics to include combinations with these new dissonant elements. The list is provided in Figure 7.

No included dissonant elements			
(01)	(02)	(06)	(048)
(01) and (02)	(01) and (06)	(01) and (048)	(02) and (06)
(01), (02), and (06)	(01, (02), and (048)	(01), (06), and (048)	(02), (06), and (048)
(01), (02), (06), and (048)			

Figure 5. Janeček's list of dissonant characteristics expanded to include all combinations of dissonant elements. The second row features the previously mentioned dissonant elements, and the next three rows show every possible combination of those dissonant elements.

17 *Změnu kvality*.

18 Uplatnění půltónových prvků, které jsou od sebe vzdáleny o půltón, a uplatnění dvou prvků tritonových, které jsou od sebe vzdáleny o půltón, představuje vskutku *zásah nového faktoru*. Je pak třeba mluvit o *půltónovém střetnutí půltónů a o půltónovém střetnutí tritonů* jako z znacích *rovnocenných s prvky samými*.

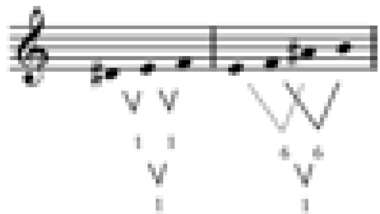

Figure 6. The two new dissonant elements Janeček describes: (012) and (0167). In the first, the interval 1 appears twice at the distance of interval 1. In the second, the interval 6 appears twice at the distance of interval 1.

No included dissonant elements					
(01)	(02)	(06)	(048)		
(01) and (02)	(01) and (06)	(01) and (048)	(02) and (06)	(02) and (012)	(01) and (0167)
(01), (02), and (06)	(01), (02), and (048)	(01), (06), and (048)	(02), (06), and (048)	(01), (02), and (012)	(01), (06), and (0167)
(01), (02), (06), and (048)	(01), (02), (048), and (012)	(01), (02), (06), and (012)	(01), (02), (06), and (0167)		
(01), (02), (06), (048), and (012)	(01), (02), (06), (012), and (0167)				
(01), (02), (06), (048), (012), and (0167)					

Figure 7. A list of all possible combinations of dissonant elements and the twenty-two different dissonant characteristics they create

Now I provide my interpretation of Janeček's taxonomic approach to chord quality in set-theoretical language. Janeček's taxonomy classifies all 350 T_n *types* into twenty-two different genera according to their inclusion of subsets (01), (02), (06), (048), (012), and (0167), or any combination of those subsets. In other words, given what I call a Janeček dissonance vector (jdv), where in each of six positions in an ordered array a 0 represents non-presence and a 1 represents presence of the respective subsets {(01); (02); (06); (048); (012); (0167)} within a given set, two sets will belong to the same genus if their jdvs are identical.

As previously mentioned, T_n *types* are all unordered sets after observing enharmonic, octave, and transpositional equivalence with a twelve-note division of the octave, as opposed to *Tn/TnI types*, which observe inversional equivalence. Both terms were introduced by John Rahn (1980). There are two simple ways to think of T_n *types*: first, they are normal order sets transposed to begin on zero; second, they are also traditional set classes, sets that are both transpositionally and inversionally equivalent, expanded to include sets that are inversionally related, like (047) and (037). T_n *types* are still unique and exhaustive like set classes, but T_n *types* are a somewhat larger group of sets: 350 T_n *types* vs 222 set classes. See Figure 8 for a visual explanation of T_n *types*.

Figure 8. An example of the relationship between T_n *types* and set classes

These T_n *types* are organized into genera according to their inclusion of the subsets (01), (02), (06), (048), (012), and (0167). A subset is any pc set that is included in some other set of higher cardinality. Any two sets in a genus will include the same subsets, regardless of quantity, from this list. See Figure 9 – sets (02357) and (01358) both include subsets (01) and (02), while not including any of the other listed subsets, and therefore are members of the same genus.

(02357) subsets	*(01358) subsets*
(01) (02) (03) (04) (05) (013) (024) (015) (025) (027) (037) (0135) (0235) (0237) (0247) (0257)	**(01) (02)** (03) (04) (05) (013) (024) (015) (025) (027) (037) (0135) (0237) (0247) (0158) (0358)

Figure 9. Lists of all subsets included in (02357) and in (01358). Both include (01) and (02), but do not contain (06), (048), (012), or (0167), and are thus members of the same genus.

My jdv, represented with curly brackets, provides a succinct way to express inclusion of the subsets listed and thus genus membership of a set. Any set has one jdv. The jdv expresses whether a set includes any of the six listed subsets. Each one of six positions represents one of those subsets. A 0 in a position represents subset exclusion, and a 1 represents subset inclusion, respective of the following order: {(01); (02); (06); (048); (012); (0167)}. Figure 10 provides three examples. Most importantly, all sets within a given genus will have the same jdv.

(02357)	jdv of {110000}	only (01) and (02) included
(0135689)	jdv of {111100}	only (01), (02), (06), and (048) included
(0134578)	jdv of {111110}	all except (0167) included

Figure 10. Three examples of Janeček dissonance vectors (jdvs)

For demonstration, Figure 11 shows that T_n *type* (037) and (05) both have a jdv of {000000} and belong to the same genus. Neither set includes subset (01), (02), (06), (048), (012), or (0167). The sets (014589) and (0148), with a jdv of {100100}, are also of the same genus. They both include subset (01) and (048) but none of the other subsets.

Genus with jdv of {000000}	*Genus with jdv of {100100}*
(05)	(0148)
(037)	(014589)

Figure 11. Two members of two different genera identified according to their jdv

We recognize that Janeček, like most theorists classifying chord quality, creates genera based on intervallic content. Many studies concerned with intervallic content, however, either implicitly or explicitly do not recognize the different affective quality of interval classes. In the case of Janeček's chord quality taxonomy based on dissonances, he is concerned only with those intervals that create dissonances. In other words, for Janeček, the variable affective quality of different interval classes is intuitively so different that he makes that apparent in a taxonomic system. Some recent theorists, like Isaacson (1996) and Samplaski (2000), have considered whether recognizing variable affective quality might be warranted. This issue, both for Janeček and more recent theorists, comes most into play when the question of perception is at hand. When it comes to creating neat and systematic taxonomies of chords, perception need not necessarily be a consideration. However, studies that make claims about a listener's ability to hear chord groupings or similarity relations must consider whether features of a taxonomy that make it neat, like the equal affective influence of intervals, produce a taxonomy that aligns with listener experience.

Now I will expand upon the Janeček dissonance vector from my interpretation. If, for a moment, we imagine that Janeček's approach only recognized ic 1, 2, and 6 as dissonant elements, we could describe his genera in the following way:

> Given two sets, they are members of the same genera if, and only if, they include the same combination of ic 1, 2, and 6, regardless of all other interval classes. For example, any set in the genera including ic 1 and ic 2 will have an interval class vector of <nmxyz0>, where n > 0, m > 0, and x, y, and z, can be any integer.

For example, sets (012) <210000> and (01234) <432100> both include ic 1 and 2 but do not include ic 6. Thus, they would be members of the same genus. However, because Janeček observes three dissonant elements whose inclusion in a set cannot be gleaned from an icv, I propose the Janeček dissonance vector (jdv) as an alternative. In the jdv, each one of six positions functions as a placeholder to represent one of the six dissonant elements, that is, dissonant subsets, in the following order: {(01); (02); (06); (048); (012); (0167)}. Inclusion of a subset is represented by a 1 in its position, and exclusion is represented by a 0. Thus, any (037) or (047) would have a jdv of {000000}: those pc sets include no dissonant elements. For another example, (0134679t) has a jdv of {111001}: it contains the dissonant elements (01), (02), (06), and (0167).

This approach does away with two features of icvs that are not relevant to Janeček's chord quality taxonomy based on dissonances: the inclusion of consonant subsets in any set and the quantity of a dissonant element beyond simply inclusion. For Janeček's taxonomy based on dissonance, consonance plays no role in determining genus. Further, his taxonomy is unconcerned with the number of unique occurrences of any subset within a set. This leaves Janeček's taxonomy with twenty-two different genera, as shown in Figure 12.

jdv of genus	*Pc sets included*	*tally*
{000000}	(03) (04) (05) (047) (037)	5
{100000}	(01) (014) (045) (015) (034) (0347) (0158) (0145)	8
{010000}	(02) (025) (027) (024) (035) (0247) (0358) (0257) (0357) (02479)	10
{001000}	(06) (036) (0369)	3
{000100}	(048)	1
{110000}	(013) (023) (0134) (0457) (0135) (0235) (0245) (0237) (01358) (02457) (02357) (03578) (024579)	13

jdv of genus	*Pc sets included*	*tally*
{101000}	(016) (056) (0367) (0156) (0147)	5
{100100}	(0148) (0348) (01458) (03478) (014589)	5
{011000}	(026) (046) (0268) (0368) (0246) (0258) (02469)	7
{111000}	(0136) (0356) (0137) (0467) (0146) (0256) (0157) (0267) (0236) (0346) (03568) (03467) (02578) (02568) (02467) (02369) (02368) (02367) (02358) (02356) (01578) (01479) (01469) (01457) (01378) (01369) (01368) (01357) (01356) (01347) (01346) (024679) (023579) (023578) (013578) (013568) (023568) (013479) (013689) (013469) (023569) (013568t)	42
{110100}	(01348)	1
{110010}	(012) (0125) (0124) (0123) (0234) (0345) (01245) (02347) (01235) (01234) (02345) (03457) (01345) (012345) (023457)	15
{101100}	(01478)	1
{101001}	(0167)	1
{011100}	(0248) (02468) (02468t)	3
{111100}	(02478) (03468) (02458) (01468) (013478) (013589) (024578) (013579) (013569) (024689) (013468) (014689) (014578) (0135689) (013468t) (0134689)	16
{111001}	(01367) (01467) (013679) (013467) (014679) (023689) (0134679) (0235689) (0134679t)	9
{111010}	(0456) (0127) (0126) (02346) (04567) (03678) (03567) (03456) (02567) (02456) (01456) (01268) (01258) (01257) (01256) (01247) (01246) (01237) (01236) (035678) (034567) (025678) (024569) (024567) (023678) (023567) (023479) (023469) (023467) (023456) (013457) (013456) (012568) (012479) (012469) (012457) (012456) (012369) (012368) (012358) (012357) (012356) (012347) (012346) (0234679) (0234579) (0235679) (0245679) (0123469) (0123457) (0234569) (0234567) (0123479) (0123568) (0235678) (0123456) (02345679)	57
{110110}	(03458) (013458) (034578)	3
{111110}	(01248) (02348) (012569) (012589) (012579) (012458) (012468) (023468) (014568) (034568) (024568) (023478) (023458) (034689) (034678) (012348) (0134578) (0124579) (0124589) (0245689) (0123569) (0234578) (0124689) (012468t) (0123579) (01345679) (0123468) (0124568) (0134568) (0234568) (0125689) (0135789) (0134589) (0245789) (0246789) (0124569) (0134569) (0123458) (0234678) (0245678) (0345678) (01345689) (01234569) (0124579t) (0123568t) (01245689) (0124568t) (0134568t) (0123468t) (01234579) (01234568) (01345789) (02456789) (02345678) (01234568t) (01245689t)	56

jdv of genus	*Pc sets included*	*tally*
{111011}	(01267) (01567) (012467) (012367) (012578) (012678) (013678) (012378) (012567) (013567) (014567) (0123567) (0123467) (0124679) (0123578) (0123678) (0123679) (0235789) (0123689) (0125678) (0135678) (0124567) (0134567) (01234679) (01235678) (01236789) (01234567) (0123578t) (02356789)	29
{111111}	(012478) (014678) (0124578) (0234689) (0135679) (0145679) (0125679) (0234589) (0123478) (0234789) (0124678) (0134678) (0145678) (01235679) (02345789) (01234678) (0124578t) (01235689) (02345689) (01234578) (0124678t) (01235789) (0134678t) (01234689) (01245679) (01345679) (01245789) (01246789) (01234789) (01346789) (01356789) (01234589) (02346789) (01245678) (01345678) (012345678) (012345679) (023456789) (013456789) (012345689) (012456789) (012345789) (012356789) (012346789) (01234579t) (01234578t) (01234689t) (01235678t) (01235679t) (01235689t) (01235679t) (012346789t) (012345789t) (012345689t) (012345679t) (012345678t) (0123456789) (0123456789t) (0123456789te)	60

Figure 12. All genera in Janeček's taxonomy, membership of each genus, and the tally of members in each genus

As I mentioned earlier, Hindemith (1937), like Janeček, used the semitone, the whole tone, the tritone, and the augmented triad to categorize chords. As we know, however, Janeček does not stop there, and proceeds to include any combination of those dissonances, as well as two special cases. The resulting difference between the two taxonomies is significant. Dirk Wingenfeld (1991) explains that of 350 possible chords in Hindemith's system, one genus contains 276 members, while the next largest genus contains only 45 and the remaining genera all contain less than 10. Janeček, on the other hand, as shown in Figure 12, has a more even distribution of membership across his genera. Admittedly, the distribution is only relatively even.

We have already discussed several features of Janeček's taxonomy that helped place it historically. These included: an exhaustive list of all possible harmonies; that list being all 350 T_n *type* sets; weighting the affective quality of intervals differently; and using dissonances as the primary element for determining genus. On these topics Janeček aligned with Bacon, expanded on some ideas of Hába and Hindemith, and took issue with the approaches of Hindemith and Schoenberg. Of further interest is the similarity between Janeček and Howard Hanson (1960). Janeček and Hanson together are an example of two geographically and communicatively isolated authors with similar achievements. Neither author was a twelve-tone composer, yet the work of both authors represents what Jonathan Bernard (1997) describes as a "proto-set theory." Both begin with the twelve-note division of the octave in equal temperament; both focus on the interval content of sonorities

– Hanson on all intervals, Janeček on dissonances; both effectively establish the concept of interval class. Several differences arise between the two of course; given Hanson's focus on complete interval content he also recognizes Z-related sets, which he calls the *isometric relation*. Janeček's approach, however, does not share this feature, but does include other features typical of set theory, including: an exploration of symmetrical and asymmetrical sets; sets and their complements; and an acknowledgment of the inversional relationship between some T_n sets. Where Janeček differs from set theory, besides not recognizing inversionally equivalent set classes, is his lack of integer notation, which would prove to inhibit some more complex features of set theory.

The historical interest in comparing Janeček to Hanson comes in determining which of the two theories proves to be the earliest proto-set-theoretical approach. Although published in 1960, according to Robert Sutton (1987), Hanson's book is the product of work dating back to the 1940s and was essentially complete in 1955. Janeček's *Základy* appears to have a similar story. According to Jaroslav Smolka (1995), portions of *Základy* were published as early as the 1940s and the book was ready to be published in 1950, but due to complications with the political climate in Czechoslovakia, the book did not see publication until 1965. I make no claim here to the chronology of either theory, besides that both Janeček and Hanson appear to be the earliest precursor of the set-theoretical tradition.

In conclusion, I have provided an interpretation of Janeček's dissonance-based chord quality taxonomy. This interpretation reveals an approach to chord classification that is novel in the particular factors that determine genus membership, namely, the subsets (01), (02), (06), (048), (012), and (0167). Additionally, this interpretation reveals that Janeček, along with Howard Hanson, proves to be one of the earliest theorists to develop a proto-set-theoretical approach. At the beginning of this study I proposed that historical music-theoretical approaches, such as Janeček's, provide current authors with material to further develop our theoretical understanding of music in the twentieth century. For Janeček's theory, this could take the shape of something like Samplaski's (2000) study of listeners' abilities to perceive chord-classification schemes ranging from Hindemith to Forte. Such a study would evaluate whether Janeček's dissonant elements do prove to be salient for listeners in determining chord quality. Admittedly, I think such a study would, just like every scheme Samplaski tested, not prove positive. Most of all because one would encounter the difficulty of testing for chord quality alone, when Janeček – like Hindemith, Krenek, and Harrison – admits that the affect of the chord is dependent on voicing. However, I would not be surprised if a modification to Janeček's taxonomy could prove fruitful in studies of perception.

With Janeček's approach there also remains the question of whether his taxonomy might be conducive to work in similarity relations, given that he does not assume, as other theories either implicitly or explicitly do, that each interval of a set plays an equal affective role in determining chord quality. Even if his particular genus determiners prove either perceptually false and theoretically unattractive, then Janeček becomes another voice to question the near ubiquity of not only the equal affective power of intervals, but also whether inversional equivalence should be a feature of chord quality taxonomies. Samplaski (2000) suggests that it might not be; Harrison (2016) chooses to recognize the difference between inversionally related sets, and historically inversional equivalence is not the norm, as in Bacon (1917) and Hindemith (1937). Perhaps effective theories of music in the twentieth and twenty-first centuries need to account for the variable affective influence of intervals and to de-emphasize inversional equivalence, as Janeček does.

References

Bacon, Ernst Lecher. "Our Musical Idiom." *The Monist* (1917): 506–607.

Bernard, Jonathan W. "Chord, Collection, and Set in Twentieth-Century Theory." In *Music Theory in Concept and Practice*, edited by James M. Baker, David W. Beach and Jonathan W. Bernard, 11–51. Rochester: University of Rochester Press, 1997.

Chrisman, Richard A. "A Theory of Axis-Tonality for Twentieth-Century Music." PhD dissertation, Yale University, 1969.

Forte, Allen. *The Structure of Atonal Music*. New Haven: Yale University Press, 1973.

Hába, Alois. *Neue Harmonielehre des diatonischen, chromatischen, viertel-, drittel-, sechstel-, und zwölftel-Tonsystems*. Leipzig: F. Kistner & C. F. W. Siegel, 1927.

Hanson, Howard. *Harmonic Materials of Modern Music: Resources of the Tempered Scale.* New York, Appleton-Century-Crofts, 1960.

Harrison, Daniel. *Pieces of Tradition: An Analysis of Contemporary Tonal Music*. Oxford Studies in Music Theory; Variation: Oxford Studies in Music Theory. New York: Oxford University Press, 2016.

Hindemith, Paul. *Unterweisung im Tonsatz*. Mainz: B. Schott's Söhne, 1937.

Janeček, Karel. "Posální Hudební Teorie." *Hudební Rozhledy* (1957): 920–22.

——. *Základy moderní harmonie*. Praha: Nakladatelství Ceskoslovenskí akademie ved, 1965.

Krenek, Ernst. *Studies in Counterpoint: Based on the Twelve-Tone Technique*. New York: G. Schirmer, Inc, 1940.

Morris, Robert D. "Generalizing Rotational Arrays." *Journal of Music Theory* 32(1) (1988): 75–132.

Quinn, Ian. "A Unified Theory of Chord Quality in Equal Temperaments." PhD dissertation, University of Rochester, Eastman School of Music, 2004.

Rahn, John. *Basic Atonal Theory*. Longman Music Series; Variation: Longman Music Series. New York: Longman, 1980.

Samplaski, Arthur G. "A Comparison of Perceived Chord Similarity and Predictions of Selected Twentieth-Century Chord-Classification Schemes, Using Multidimensional Scaling and Cluster Analysis." PhD dissertation, Indiana University, 2000.

Smolka, Jaroslav. *Karel Janeček, Český Skladatel a Hudební Teoretik*. Praha: Hudební fakulty AMU, 1995.

Sutton, Robert. "Howard Hanson, Set Theory Pioneer." *Sonus* (Fall 1987): 17–39.

Wingenfeld, Dirk. "Hindemiths Akkordbestimmung Als Grundlage Für Eine Differenziertere Akkordklassifikation." *Hindemith-Jahrbuch* (1991): 110–40.

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